ESSE QUAM VIDERI

Highsted

Grammar School

KS4 Maths Knowledge Organiser

Unit 1: Number

Number

Number Problems

A Factorial is the result of multiplying a sequence of descending integers.

4! = 4 x 3 x 2 x 1

Estimation

To estimate you need to be confident with rounding and significant figures.

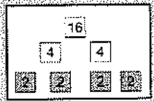
Estimate 0.456 x 145 by rounding to 1 significant figure.

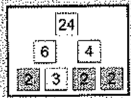
0.5 x 100 = 50

HCF and LCM

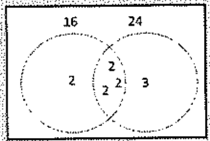
Find the HCF and LCM of 16 and 24

Step 1: Express each number as a product of its prime factors.





Put the prime factors into Venn diagram



HFC = product of the intersection: 2 x 2 x 2 = 8

LCM = product of all the numbers 2 x 2 x 2 x 2 x 3 =48

Standard Form

Writing numbers in standard form

Numbers in standard form always have to be bigger than 0 and smaller than 10

Example: Write 124, 500, 000 in standard form

$$1.245 \times 10^8$$

Example: Write 0.005678 in standard form

$$5.678 \times 10^{-3}$$

Multiplying and Dividing in Standard Form

$(2.1 \times 10^3) \times (3 \times 10^4)$

Multiply the numbers and add the indices together

2.1 x 3 = 6.3 3+4 =7

$$6.3 \times 10^7$$

$(9 \times 10^3) \div (3 \times 10^4)$

Divide the numbers together and subtract the indices

9/3 = 3 3—4 = -1

$$3 \times 10^{-1}$$

Adding and Subtracting in Standard Form

You have to change them back into normal numbers.

$$2.1 \times 10^4 + 3.2 \times 10^2 =$$
$$21000 + 320$$
$$= 21320$$
$$= 2.132 \times 10^4$$

Indices

Basic Rules of Indices

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^5}{a^3} = a^2$$

$$(a^2)^3 = a^6$$

$$a^1 = a$$

$$a^0 = 1$$

Base — y^6 — Index

$$y^6 = y \times y \times y \times y \times y \times y$$

Fractional Rules of Indices

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

If a^{-b} then we write as $\frac{1}{a^b}$

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$$

Surds

A surd is an irrational number. It doesn't terminate (stop) or repeat.

A surd is written with a square root sign: $\sqrt{2}$

Simplifying a surd

Simplify $\sqrt{200}$

Step 1: Find two factors of 200 one must be the biggest square number you can find!

$$\sqrt{100} \times \sqrt{2}$$

The root 100 simplifies to 10 and the multiplication sign disappears (Like in algebra) so you are left with:

$$10\sqrt{2}$$

Multiplying Surds

To multiply surds you just multiply the numbed under the square root sign together

$$\sqrt{3} \times \sqrt{7} = \sqrt{21}$$

For more complicated examples you must multiply the numbers first and then the surds

$$2\sqrt{3} \times 4\sqrt{7} = 8\sqrt{21}$$

Rationalising the denominator

$$\frac{4+\sqrt{5}}{\sqrt{5}}$$

To rationalise the denominator you have to remove the surd from the denominator.

You do this by multiplying numerator and denominator by the surd

$$\frac{4+\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{4\sqrt{5}+5}{5}$$

Unit 2: Algebra

Basic Algebra

Algebraic Indices

Simplify

$8qr^2 \times 3qr = 24q^2r^3$

You must use the rules of indices when simplifying with algebra

Solving Equations

Solve $3(x+5) = 21$

Expand first

$3x + 15 = 21$

Solve for x

$3x = 6$

$x = 2$

Substitution

When we substitute values into a formula we take out the variables and put in the numbers.

Example: 2a + 4b

Where a = -3 and b = 5

You do 2 x -3 = -6

And 4 x 5 = 20

Then add them together:

-6 + 20 = 14

Expanding

Expanding Single Brackets

Expand $3(x + 4 + y)$

Multiply in grid method

x	x	4	y
3	$3x$	12	$3y$

$3(x + 4 + y) = 3x + 12 + 3y$

Expanding double brackets that look like single brackets

Expand $(x+1)^2 = (x+1)(x+1)$

Multiply in grid method

x	x	1
x	x^2	x
1	x	1

$= x^2 + x + x + 1 = x^2 + 2x + 1$

Expanding double brackets

Expand $(x + y)(x + y)$

Multiply in grid method

x	x	y
x	x^2	xy
y	xy	y^2

$= x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

When expanding brackets it is

easier to use grid method.

Make sure you simplify at the end

Factorising

Factorising Single Brackets (Numbers)

Factorise $10x + 15$.

Find the HCF of the numbers. → $10x + 15$ ← Divide each term by the HCF and close the bracket.

$= 5(2x + 3)$

HCF = 5

Factorising into a single bracket

Variables and numbers

Factorise $2ab + 4b$.

Find the HCF of the variables. → $2ab + 4b$ ← Divide each term by the HCFs and close the bracket.

$= 2b(a + 2)$

HCF = 2

HCF = b

Only 'open the brackets' once all HCFs are found.

Factorising Double Brackets

Factorise the following quadratic expression into double brackets.

$x^2 + 9x + 18$

$= (x + 6)(x + 3)$

Factors of 18

1, 18

2, 9

3, 6

Write a list of factor pairs of the constant term.

Choose the pair that add to make "+9".

You can put these in either bracket!

Why must the factor pair be... ?

positive × positive

negative × negative

Sequences

Linear Sequence

Find the nth term of the following sequence:

14, 12, 10, 8, 6 ….

-2 -2 -2 -2 = -2n

To find the constant we find the term before The 1st term which is 16

The nth term is = -2n + 16

Quadratic Sequence

Example: Find the formula for the nth term of the sequence:

2, 5, 10, 17

Find the differences

2 5 10 17

+3 +5 +7

+2 +2

Not constant so find second differences

This is constant

The second difference is CONSTANT so the formula for the nth term must contain a n^2. The number in front of n^2 is half the constant difference.

n^2

Term number	1	2	3	4
Sequence	2	5	10	17
	+1	+1	+1	+1
n^2	1	4	9	16

This is constant so now we can work out the nth term

$n^2 + 1$

Unit 3: Graphs

Linear Graphs

All straight line graphs are in the form:
$y = mx + c$

$y = \boxed{m}x + \boxed{c}$

Gradient — y intercept

The gradient is a measure of how <u>steep</u> the graph is

The y - intercept is which number on the y axis the graph goes through

Formula for finding the gradient

$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{y - y_1}{x - x_1}$$

Finding the gradient

Pick two points on the line

Label your first point (x and y)

Label your second point (x_2 and y_2)

Substitute the points into the formula

Finding the equation of a line form two points

A line passes through the points (4,7) and (8,15). Find the equation of the line.

Gradient = $\frac{y-y_1}{x-x_1}$

Gradient = $\frac{7-15}{4-8} = -\,-\frac{8}{-4} = -2$

$y = mx + c$

$y = -2x + c$

$7 = -2(4) + c$

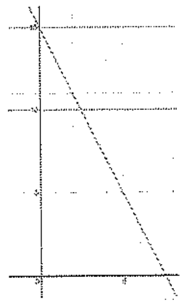
$7 = -8 + c$

$15 = c$ $\qquad y = -2x + 15$

Step 1: Find the gradient

Step 2 : Pick a point

Step 3 : Use gradient and point to find the equation

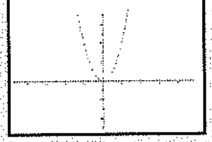


The coordinates of a midpoint of a line segment is

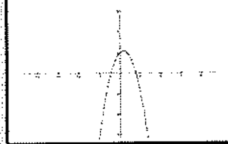
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Graphs

Positive Quadratic Graphs have a U shape

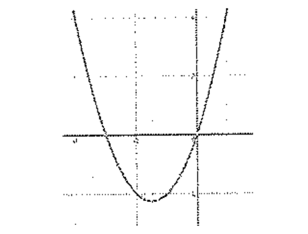


Negative Quadratic Graphs have a ∩ shape



Plot the graph of $y = x^2 + 3x$

x	-4	-3	-2	-1	0	1	2
$y = x^2 + 3x$	4	0	-2	-2	0	4	10



Finding the roots of quadratic graphs

You can find the roots of a quadratic graphs by factorising

Sketch the graph for this equation

$y = x^2 + 4x$

$$y = x(x + 4)$$

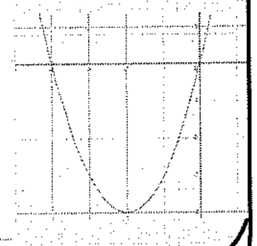
Step 1 : Factorise

Step 2 : Set each bracket to zero

Step 3 : Solve to find your roots

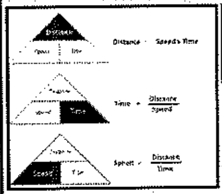
Step 4, set x equal to zero to find the y intercept

Step 5, ∩ or U shape?



Travel Graphs

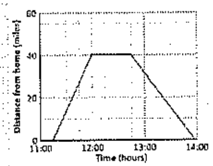
Speed Distance Time Graphs



A distance-time graph plots the distance an object travels against the time it takes it to travel.

This the gradient of the line and tells us the <u>rate of change</u> of distance with respect to time.

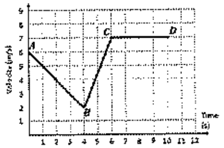
For a distance-time graph, the rate of change is the speed.



Velocity Time Graphs

Velocity is defined as the rate of travel of an object, along with its direction. Velocity tells you how fast an object is moving as well as in what direction it is moving.

The <u>gradient</u> of a velocity time graph represents acceleration



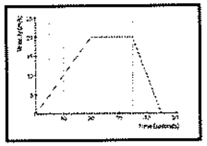
Determine the acceleration from:

a) A to B: $-4 \div 4 = -1\ m/s^2$

b) B to C: $5 \div 2 = 2.5\ m/s$

c) C to D: $0\ m/s^2$

The <u>area</u> under a velocity time graph represents distance travelled



Find the total distance travelled.

Using whole trapezium:

$$\frac{15 + 45}{2} \times 20 = 600\ m$$

Cubic, Reciprocal and Circular Graphs

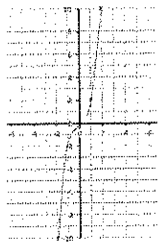
Cubic Graphs

Cubic equations are in this form

$y = ax^3 + bx^2 + cx + d$

$y = x^3 + x$

x	-2	-1	0	1	2
y	-10	-2	0	2	10

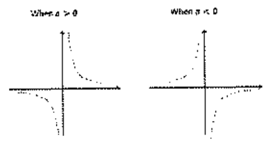


Reciprocal Graphs

A reciprocal equation will be in this form

$$y = \frac{a}{x}$$

a is a constant while x is a variable, so we might have $y = \frac{3}{x}$



The lines $x = 0$ and $y = 0$ are called asymptotes. An asymptote is a straight line which the curve approaches at infinity.

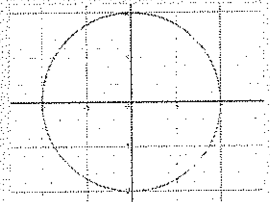
Equation of a Circle

The equation of a circle can be expressed in the form

$$x^2 + y^2 = r^2$$

Where r is the radius

$$x^2 + y^2 = 4$$

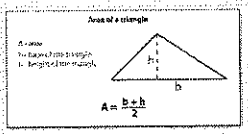


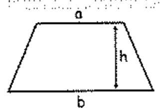
Unit 4: Area and Volume

Area

Formulas

area = base x height



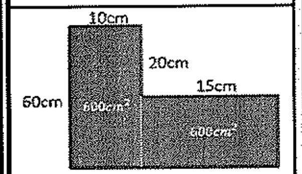


Area of Trapezium $= \frac{1}{2}h(a+b)$

Compound Area

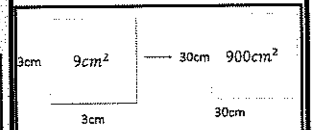
Find the area of the Compound shape

1) Split the shape up
2) Find any missing sides
3) Calculate the area of both shapes
4) Add the areas together



$600cm^2 + 600cm^2 = 1200cm^2$

Converting Units



If the length increases by a scale factor k, the area increases by this squared, i.e. k^2

Convert $5m^2$ to cm^2

1m = 100cm

$100^2 = 10{,}000$

$5 \times 10{,}000 = 50{,}000cm^2$

Steps:

Find the conversion scale factor

Square it

Multiply by the original

Bounds

Bounds

The upper bound of a number is the highest value before rounding.

The lower bound of a number is the lowest value before rounding

A plank of wood is 2.4cm to one decimal place

Find the upper and lower bound

Lower Bound = 2.35 Upper Bound = 2.45

Bounds always end in 5

Multiplying with Bounds

The upper bound of a multiplication is always the two upper bounds multiplied together

The lower bound of a multiplication is always the two lower bounds multiplied together

Dividing with Bounds

The upper bound of a fraction is always

$$\frac{\text{Upper bound of the numerator}}{\text{Lower Bound of the denominator}}$$

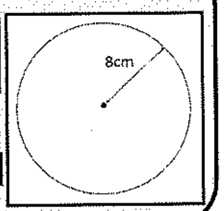
The lower bound of a fraction is always

$$\frac{\text{Lower bound of the numerator}}{\text{Upper Bound of the denominator}}$$

Circles

Area of a circle

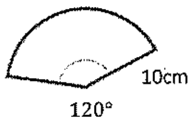
The formula for the area of the circle is πr^2



$8^2 \times \pi = 201.1cm^2$

Area of a sector

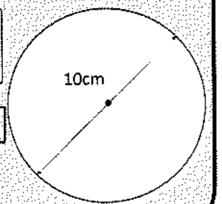
$\frac{\theta}{360} \times \pi r^2$



$\frac{120}{360} \times 100\pi = 104.7cm^2$

Circumference of a circle

The formula for the circumference of the circle is πd

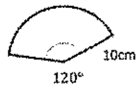


$10 \times \pi = 314cm$

Arc Length

$\frac{\theta}{360} \times \pi d$

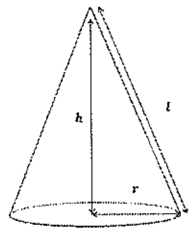
Calculate the length of the arc



$\frac{120}{360} \times 20\pi = 20.9cm$

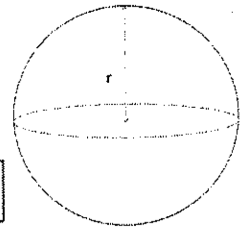
Pyramids, cones, cylinders & spheres

Cone



$Volume = \frac{1}{3}\pi r^2 h$

Spheres

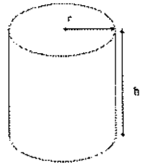


Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

Cylinder

The formula for the volume of a cylinder is $\pi \times r^2 \times h$

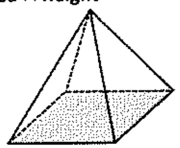


The formula for the surface area of a cylinder is

$A = 2\pi r h + 2\pi r^2$

Pyramid

Volume of Pyramid = ⅓ × base area × height



Unit 5: Equations and Inequalities

Solving Quadratic Equations

Solving by factorising

Solve the equation : $12x^2 - 14x = 0$

$2x(6x - 7) = 0$

Set each part equal to 0

$2x = 0$ $\quad$ $6x - 7 = 0$

$x = 0$ $\quad$ $+7 \; +7$

$6x = 7$

$\div 6 \; \div 6$

$x = \frac{7}{6}$

Solve $x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x - 3 = 0$ $\quad$ $x - 4 = 0$

$+3 + 3$ $\quad$ $+4 + 4$

$x = 3$ $\quad$ $x = 4$

Solve: $4x^2 - 9 = 0$

$(2x+3)(2x-3) = 0$

$2x + 3 = 0$ $\quad$ $2x - 3 = 0$

$x = -\frac{3}{2}$ $\quad$ $x = \frac{3}{2}$

Steps:

Factorise the equation

Set each bracket equal to 0

Solve for x

Solving by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) Compare with $ax^2 + bx + c$ to Identify a, b and c

2) Substitute a, b and c into the quadratic formula.

3) Simplify the three sections.

4) Split into two and solve.

Solve the following quadratic equation, giving your answer to two decimal places.

$4x^2 - 10x - 7 = 0$

$a = 4$ $\quad$ $b = -10$ $\quad$ $c = -7$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{10 + \sqrt{212}}{8} \quad x = \frac{10 - \sqrt{212}}{8}$$

$x = 3.07$ (2dp) $\quad$ $x = -0.57$ (2dp)

Simultaneous Equations

Solving by Elimination

Steps:

1) Make the x's or the y's the same using LCM

2) Label the equations 1 and 2

3) If the signs are the same subtract if not add

4) Isolate the variable by dividing

5) Find the unknown variable by substituting the known variable into one of the 2 equations.

$2x + 5y = 24$

$4x + 3y = 20$

x2

LCM of 2 and 4 is 4

$4x + 10y = 48$ ①

$4x + 3y = 20$ ②

$7y = 28$

$\div 7 \quad \div 7$

$y = 4$

$4x + 3(4) = 20$

$4x + 12 = 20$

$-12 \quad -12$

$4x = 8$

$\div 4 \quad \div 4$

$x = 2$

$$x = 2, y = 4$$

Solving by Substitution

$3x - 2y = 0$ ①

$2x + y = 7$ ②

$y = 7 - 2x$ ②

$3x - 2(7 - 2x) = 0$

$3x - 14 + 4x = 0$

$7x - 14 = 0$

$7x = 14$

$x = 2$

$2(2) + y = 7$

$4 + y = 7$

$y = 3$

$$x = 2, y = 3$$

Step 1: Rearrange the equation for x or y

Step 2: Substitute the equation into the other equation

Step 3: Solve the equation for x or y

Step 4: Substitute the value into the first equation

Completing the Square

Completing the square

Express the following quadratic expression in the form $(x + p)^2 + q$.

$x^2 - 4x + 10$

$\frac{4}{2} = 2$

$(x-2)^2$

$(x-2)^2 - 2^2$

$(x-2)^2 - 4$

$(x-2)^2 - 4 + 10$

$(x-2)^2 + 6$

1) Halve the coefficient of x

2) Write the halved coefficient inside a bracket and square it

3) Square the halved coefficient and subtract it outside the bracket

4) Add the coefficient to the end and simplify

Turning Points

$(x-2)^2 + 6$

The completed square form shows the turning point.

The minimum point of this equation is (2, 6)

Solving by completing the square

Solve the equation by completing the square

$x^2 - 8x - 2 = 0$

$(x-4)^2 - 18 = 0$

$+18 \quad +18$

$(x-4)^2 = 18$

$\sqrt{} \quad \sqrt{}$

$x - 4 = \pm\sqrt{18}$

$+4 \quad +4$

$x = 4 \pm \sqrt{18}$

$x = 4 + \sqrt{18}$ or $x = 4 - \sqrt{18}$

Steps:

1) Get the equation into completed square form

2) Isolate the bracket on one side

3) Square root both sides

4) Isolate the x on one side

Inequalities

Solving linear inequalities

$4x + 7 > 35$

$-7 \quad -7$

$4x > 28$

$\div 4 \quad \div 4$

$x > 7$

$-4x + 7 > 35$

$-7 \quad -7$

$-4x > 28$

$\div -4 \quad \div -4$

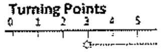
$x < -7$

Inequalities on a number line

x > 3 — This is known as a 'strict' inequality.

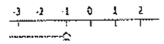
Means: x is (strictly) greater than 3.

Turning Points



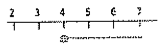
x < -1

Means: x is (strictly) less than -1.



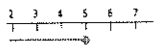
x ≥ 4

Means: x is greater than or equal to 4.



x ≤ 5

Means: x is less than or equal to 5.



Solving Quadratic inequalities

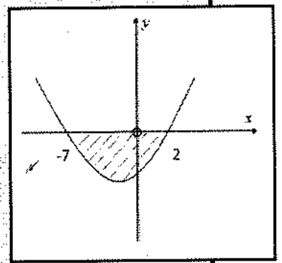
Solve $x^2 + 5x - 14 < 0$

$(x+7)(x-2) < 0$

$(x+7) > 0$ $\quad$ $(x-2) < 0$

$x > -7$ $\quad$ $x < 2$

$-7 < x < 2$



Steps:

1) Factorise

2) Sketch the curve

3) Shade the area that satisfies the inequality

4) Write the inequality

Unit 6: Angles and Trigonometry

Angle Facts

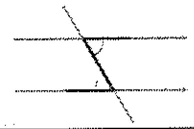
Basic Angle Facts

- Angles on a straight line sum to 180°
- Angles around a point sum to 360°
- Interior angles of a triangle sum to 180°
- Isosceles triangles have two sides the same and two base angles the same
- Equilateral triangles have the same sides and angles
- Interior Angles of Quadrilaterals sum to 360°

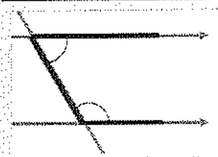
Angles in Parallel Lines

Vertically Opposite Angles are equal

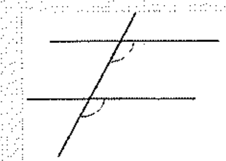
Alternate Angles are equal



Co-interior Angles sum to 180°



Corresponding Angles are equal



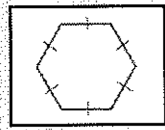
Interior and Exterior An-

Interior Angles

The sum of the interior angles of any polygon with n sides is (n - 2) x 180 °.

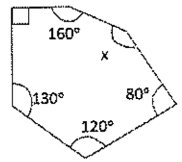
n = amount of sides

Find the sum of the interior angles of this regular Hexagon



=(n - 2) x 180
=(6 - 2) x 180
= 4 x 180 =
720°

Find the value of angle x



Sum of all angles = 4 x 180 = 720°

Sum of known angles =
90 + 160 + 130 + 120 + 80
= 580°

720° - 580° = 140°

Exterior Angles

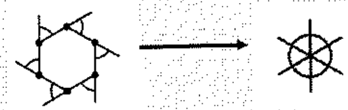
The sum of the exterior angles of a polygon will add up to 360°

An interior angle and exterior angle on a straight line will always add up to 180°

To find the size of one exterior angle of a regular polygon when given the sides we will need this formula

$$\frac{360}{n}$$

Where n is the amount of sides



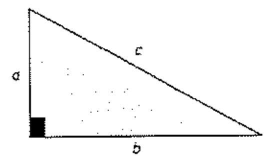
Pythagoras

The formula

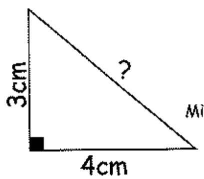
In any right-angled triangle the square of the hypotenuse is equal to the sum of the squares on the other two sides

In other words:

$a^2 + b^2 = c^2$

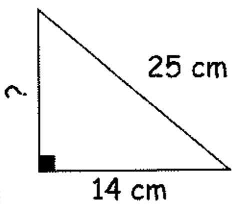


Finding a hypotenuse



$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$5 = c$
Missing side =5cm

Finding a shorter side



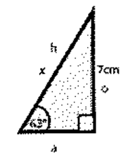
$a^2 = c^2 - b^2$
$a^2 = 25^2 - 14^2$
$a^2 = 625 - 196$
$a^2 = 429$
$a = \sqrt{429}$
$a = 20.7$
Missing side = 20.7cm

Trigonometry

Finding a missing side

Steps

1) Label the triangle
2) Work out which trig formula you need
3) Substitute the angle And the sides into the formula
4) Rearrange the equation To isolate the missing side
5) Put the numbers into your calculator



$\sin(62) = \frac{7}{x}$

$x\sin(62) = 7$

$x = \frac{7}{\sin(62)}$

$x = 7.9cm$

$$\sin(x) = \frac{o}{h}$$

$$\cos(x) = \frac{a}{h}$$

$$\tan(x) = \frac{o}{a}$$

Finding a missing angle

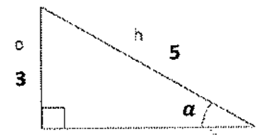
1) Label the sides

2) Decide whether you are using Sin Cos or Tan

$\sin(x) = \frac{o}{h}$ | $\cos(x) = \frac{a}{h}$ | $\tan(x) = \frac{o}{a}$

3) Put the values into the formula

4) Use the inverse trig button to find The size of the angle



$\sin(a) = \frac{3}{5}$ $\quad a = \sin^{-1}\left(\frac{3}{5}\right)$

$a = 36.9°$

Unit 7: Transformations and Constructions

Transformations

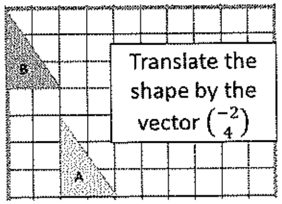
Vectors and Translations

We perform translations using vectors

A vector is a quantity that has both **magnitude** (size) **and direction**.

$\begin{pmatrix} x \\ y \end{pmatrix}$

X = number of moves to the right or left
Y= number of moves up or down



Vector arithmetic

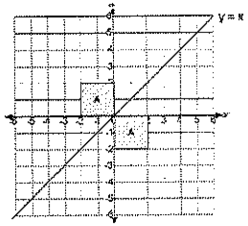
$$\binom{5}{-2} + \binom{-1}{-2} = \binom{4}{-4}$$

Reflections

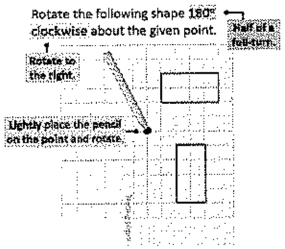
Reflect the shape in the equation y = x

Plot the line y = x

Reflect the shape in the line



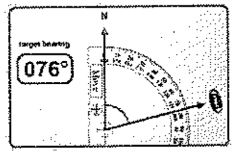
Rotations



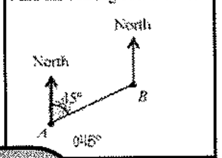
Bearings

Bearings checklist

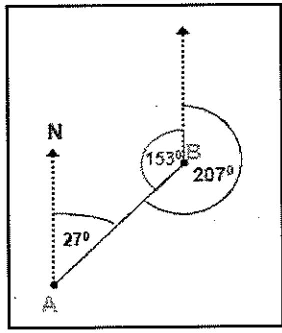
1. Measured FROM
2. Measured from North.
3. In a clockwise direction.
4. Written as 3 figures.



Find the bearing of *B* from *A*



My bearing from A to B is 027°. But what is my bearing from B to A?

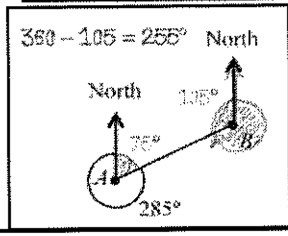


180 – 27 = 153
(co-interior angles sum to 180°)

360 – 153 - 207°
(angles around a point sum to 360°)

A from B = 207°

Find the bearing of *A* from *B*



Enlargement

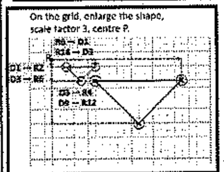
An **enlargement** is a type of transformation that involves making a shape larger or smaller.

Enlargement by a positive integer scale factor

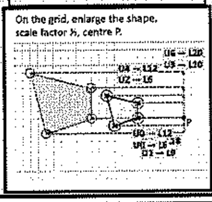
Steps:

1) Draw a point on each Vertex
2) Count the squares from the point to each vertex (do this one at a time)
3) Multiply the distance by the Scale factor
4) Draw on the new points and Join them up

Integer Scale Factor



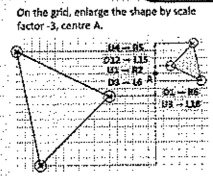
Fractional Scale Factor



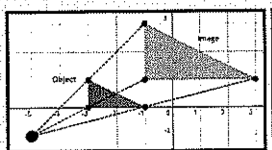
Enlargement by a negative scale factor

To **enlarge a shape by a negative scale factor**, we first **describe the movement from the centre** to a **point on the shape**.

Then we **reverse the movements starting from** the **centre using the given scale factor**.



Describing an enlargement

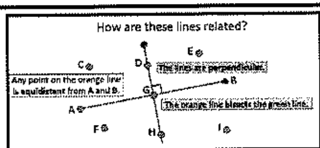


Enlargement of scale factor 2 About the point (-5, -1)

Steps:

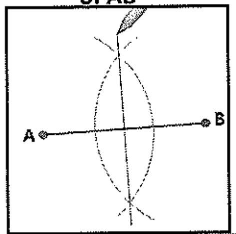
1) Draw a point on each Vertex
2) Join up each point by A straight line
3) The point where all the Straight lines meet is the Point of enlargement

Constructions and Loci



Never erase your construction lines!

Construct the **perpendicular bisector of AB**

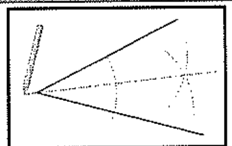


(1) Draw two equal arcs.
(2) Connect the intersections with a straight line.
(3) This line is the **perpendicular bisector** and contains all the points **equidistant** from A and B.

Angle Bisector

Draw an acute angle on your page. Construct its angle **bisector**.

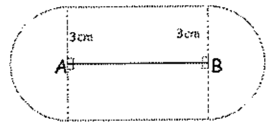
(1) Draw an arc from the vertex.
(2) Draw two more equal arcs from the intersections.
(3) Join the new intersection up to the vertex.
(4) This line is the **angle bisector** and contains all points **equidistant** from both arms of the angle.



A **locus of points** is a set of points satisfying a certain condition.

Find the locus of points 3cm from AB

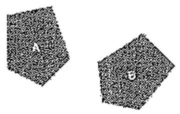
- Draw a straight horizontal line
- Open your compass to 3cm
- Point the compass on the point A and draw an arc
- Point the compass on the point B and draw an arc
- Join them together with straight lines



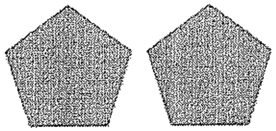
Congruence

If two shapes are congruent they are exactly the same size, they have the same sized lengths and angles.

The symbol for congruence is ≅, so we would say that $A \cong B$.

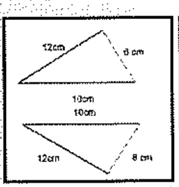


- Congruence is **symmetric**: if $X \cong Y$ then also $Y \cong X$.
- Congruence is **reflexive**: any shape is congruent to itself. For example, $A \cong A$.
- Congruence is **transitive**: if $X \cong Y$ and $Y \cong Z$ then also $X \cong Z$.



Congruency rule 1: SSS

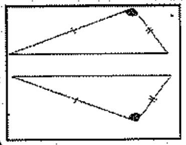
Congruency Criteria: All sides are the same.



The three sides of the triangles are the same, therefore they are congruent

Congruency rule 1: ASA

Congruency Criteria: 2 sides and one angle are the same.



The triangles have 2 sides the same length and an angle the same size, therefore they are congruent.

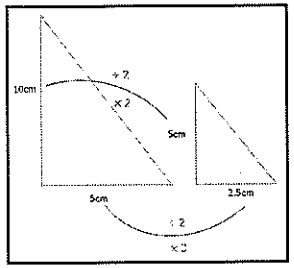
Similarity

Shapes are similar if:

They are the same shape

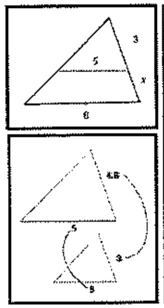
They have the same sized angles

Each length is directly proportional to the length of the corresponding shape



The yellow triangle Is twice as large as The green triangle. The ratios of the lengths are the same, therefore the triangles are similar.

Finding a missing side in a similar shape



Steps:

1) Split the shapes into two
2) Highlight the corresponding sides
3) Divide the longer side by the shorter side (this gives you the scale factor)

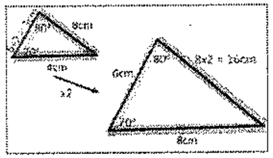
$8 \div 5 = 1.6$

4) Multiply the smaller side by the scale Factor to find the missing side

$3 \times 1.6 = 4.8$

5) Subtract 3 from 4.8 to get x

$4.8 - 3 = 1.8$

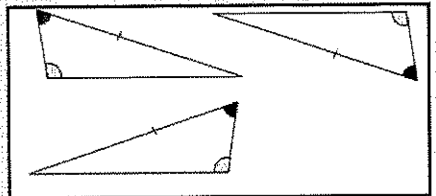


Although similar shapes can have different lengths, the angles must stay the same.

Unit 8: Similarity and Congruence

Congruency rule 3: ASA

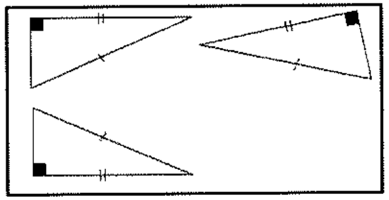
Congruency Criteria: Two angles the same size and one side the same length.



The triangles have 2 angles that are the same size and one side that is the same length, therefore the triangles are congruent.

Congruency rule 4: RHS

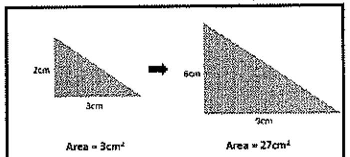
Congruency Criteria: The two triangles share a right angle, the same length hypotenuse and one side the same length.



The triangles have a right angle, the same length Hypotenuse and a side the same length, Therefore they are congruent.

Congruency

Similarity with Area



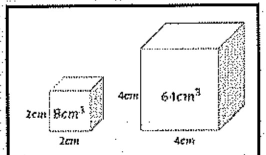
Length increased by a factor of 3

Area increased by a factor of 9

We can see that if the length increases by a scale factor k, the area increases by this squared, i.e. k^2.

	Shape X		Shape Y
Length:	2m	× 5 →	10m
Area:	$3m^2$	× 25 →	$75m^2$

Similarity with Volume



Length increased by a factor of 2

Area increased by a factor of 8

We can see that if the length increases by a scale factor k, the volume increases by this cubed, i.e. k^3.

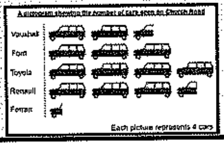
	Shape A		Shape B
Length:	6cm	× 3 →	18cm
Area:	$30cm^2$	× 9 →	$270cm^2$
Volume:	$80cm^3$	× 27 →	$2160cm^3$

Similarity with area and volume

Unit 9: Data

Diagrams

A **pictogram** uses pictures to show quantities of data.



Stem and Leaf

Stem and Leaf checklist:

1) Numbers in ascending order
2) Line separating the units
3) Key

Males		Females
7 3	2	2 4 7 9
9 6 2 1 0	3	1 2 5 5 7 9
8 8 7 5 3 2	4	0 4 4 7
9 2 1 0	5	2 7 9

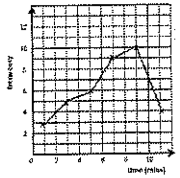
Key 36 years = 6|3 Key 36 years = 3|6

Frequency Polygons

Frequency polygons are plotted at the **mid-point.**

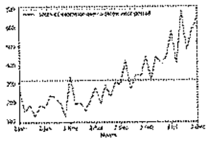
They are joined by straight lines

		Midpoints
$0 \le t < 2$	3	1
$2 \le t < 4$	5	3
$4 \le t < 6$	6	5
$6 \le t < 8$	9	7
$8 \le t < 10$	10	9
$10 \le t < 12$	4	11



Time Series

Time series graphs are line graphs plotted over time to show **trends.**



Averages

Mean

To find the mean add up all the numbers and divide by the amount.

Median

To find the median order the numbers and find the middle value

Mode

The mode is the most common value

Range

The range is the biggest value subtract the smallest

Mean from Frequency tables

Mean from a frequency table:

1) Find the total frequency
2) Multiply the first two columns together
3) Find the total of the f(x) column.
4) Divide the total f(x) by the total frequency

Goals Scored	Frequency	F x GS
0	2	0 x 2 = 0
1	5	1 x 5 = 5
2	3	2 x 3 = 6
3	8	3 x 8 = 24
4	2	4 x 2 = 8
Total	20	43

Total of all items= 43 Number of items = 20

Mean = 43 ÷ 20 = 2.15

Median from Frequency Tables

Finding the Median:

1) Find the cumulative frequency for the data
2) Add 1 to the total cumulative Frequency, then divide by 2.
3) Find where the median value lies

SCORE (X)	FREQUENCY (f)	Cumulative frequency
1	10	10
2	11	21
[redacted]		
4	9	45
5	12	57
Total	57	

Median value = $\frac{n+1}{2} = \frac{57+1}{2} = 29$ The median is 3

Pie Charts

Step 1: Find the total of the <u>frequency</u> column

Step 2: Divide 360 by the total frequency

$360° \div 30 =$
12 *degrees per person*

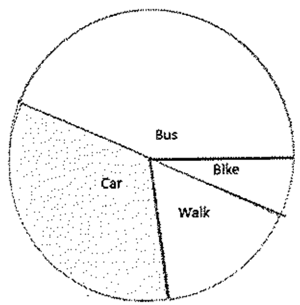
Step 3: Multiply each frequency by 12

Step 4: Draw your Pie Chart

A class was asked how they got to school:

Bus	13	156°
Car	10	120°
Bike	2	24°
Walk	5	60°
Total	30	360°

Draw a pie chart to <u>represent</u> this data



Averages

Mean from Grouped frequency tables

1) To find the mean first find the total of the frequency.

2) Find the mid points.

3) Multiply the frequency and the mid point.

4) Divide the total frequency x mid point by the total frequency.

Height cm	Frequency (f)	Mid-Value(x)	Group Total(fx)
5-15	6	10	6x10=60
15-25	4	20	80
25-35	15	30	450
35-45	3	40	120
45-55	2	50	100
Total	30		810

Estimated mean $= \frac{810}{30}$

$= 27$

Median from Frequency Tables

Finding the Median:

1) Find the cumulative frequency for the data
2) Add 1 to the total cumulative Frequency, then divide by 2.
3) Find where the median value lies

Number of phones (x)	Frequency (f)	Cumulative Frequency
0-4	5	5
5-9	8	13
10-14	4	17
15-19	9	26
20-24	3	29

The middle data item will be: $\frac{n+1}{2}$

$\frac{29+1}{2} = 15$

Therefore we know that that the median lies in the group 10- 14

Unit 10: Fractions, Ratios and Percentages

Fractions

Improper Fractions and Mixed Numbers

1) How many times the denominator goes into the numerator is the whole number
2) The numerator is the remainder
3) The denominator stays the same

Convert $\frac{15}{4}$ into a mixed number

$3\frac{3}{4}$

1) Multiply the denominator by the whole number, then add on the numerator.
2) The denominator stays the same

Convert $4\frac{6}{7}$ into an improper fraction

$4 \times 7 = 28$
$28 + 6 = 34$

$4\frac{6}{7} = \frac{34}{7}$

Adding and Subtracting Fractions

$\frac{5}{4} - \frac{1}{7}$

LCM of 4 and 7 = 28

$\frac{35}{28} - \frac{4}{28} = \frac{31}{28}$

1) Find the LCM of the denominator
2) Cross Multiply each fraction
3) Simplify

Dividing Fractions

1) Keep the first fraction
2) Change the sign to a multiply
3) Find the reciprocal of the second fraction
4) Multiply the numerators
5) Multiply the denominators
6) Simplify if needed

$$\frac{2}{3} \div \frac{3}{5} =$$

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$$

Multiplying Fractions

$$\frac{3}{4} \times \frac{7}{9} = \frac{3 \times 7}{4 \times 9} = \frac{21}{36}$$

When you multiply fractions multiply The numerators and the denominators together

Ratio

Simplifying Ratios

When simplifying ratios we must find the HCF of each number and divide.

Each number must be an integer.

Simplify the ratio

36 : 144

÷ 36 ÷ 36

1 : 4

Sharing into a ratio

Alan and Ben share £48 in the ratio 2 : 1

How much does each person get

Steps:

Add the ratios together

Divide the total amount by the ratio total

Multiply the answer by each ratio

2 + 1 = 3

48 ÷ 3 = 16

16 x 2 = £32

16 x 1 = £16

Ratio: Difference Given

Jo and Bill share money in the ratio 4:7. Bill receives £21 more than Jo. How much money was shared out?

1) Find the difference between the ratios
2) Divide the amount by the difference.
3) Multiply the answer by each original ratio

7 − 4 = 3

21 ÷ 3 = 7

7 x 4 = £28

7 x 7 = £49

28 + 49 = £77

1 : n

When writing ratios in the form 1 : n or n : 1 you must divide to make one of the numbers one.

Here the numbers can be decimals

Writing ratio in the form 1 : n

÷5 (5 : 8) ÷5

1 : 1.6

Percentages

Increasing and Decreasing by percentages

To increase by a percentage we use multipliers.

To find the multiplier we divide the percentage by 100 and add it to 1.

Increase 30 by 16%

30 x 1.6 = 34.8

To decrease by a percentage we use multipliers.

To find the multiplier we divide the percentage by 100 and subtract it from 1

Decrease 70 by 19%

70 x 0.81 = 56.1

Percentage Change

$$\text{Percentage change} = \frac{change}{original\ value} \times 100$$

What is the percentage increase from 12 to 18?

$$\frac{6}{12} \times 100 = 50\%$$

Original Value after Percentage Change

The normal price of a television is reduced by 30% in a sale. The sale price of the television is £350. Work out the normal price of the television.

Method :

÷ 7 (70% is £350) ÷ 7

10% is £50) × 10

100% is £500

Financial Maths: Interest

Depreciation

I buy a car for £20000.

It depreciates at a rate of 4% *per annum*

What will it be worth after 3 years?

Initial amount= £20000
Depreciation rate = 4%
Multiplier is × 0.96
It depreciated for 3 years

$20000 \times 0.96^3 = £17694$

Compound Interest

$\text{Initial amount} \times (1 + \text{the rate of interest})^{years}$

You save £2000 in a savings account for 4 years. The interest rate is 0.6% per annum.

Initial amount= 2000
Interest rate = 0.6%
Multiplier is × 1.006
It is in the bank for 4 years

2000×1.006^4
$= £2048.43$

Unit 11: Sine & Cosine Rules

Trigonometry with bounds

The upper bound of a multiplication is always the two upper bounds multiplied together

The lower bound of a multiplication is always the two lower bounds multiplied together

The upper bound of a fraction is always

$$\frac{\textit{Upper bound of the numerator}}{\textit{Lower Bound of the denominator}}$$

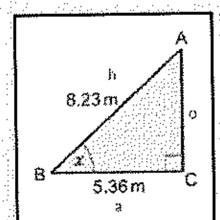
The lower bound of a fraction is always

$$\frac{\textit{Lower bound of the numerator}}{\textit{Upper Bound of the denominator}}$$

In this diagram, the measurements are correct to 3 significant figures.

a. Find the upper and lower bounds for the value of x, to 3 decimal places.

b. Give the value of x to a suitable level of accuracy.



UB of h = 8.235
LB of h = 8.225

UB of a = 5.365
LB of a = 5.355

Steps:

1) Find the upper bound And lower bound of the sides

2) Find the upper and lower Value of x using trigonometry

3) Round both values to the Nearest degree to find a Good estimate for x

$\cos x_{ub} = \frac{5.365}{8.225}$

$\cos x_{lb} = \frac{5.355}{8.235}$

$x_{ub} = \cos^{-1}\frac{5.365}{8.225} = 49.438°$

$x_{lb} = \cos^{-1}\frac{5.355}{8.235} = 49.286°$

$x = 49°$ (to the nearest degree)

Sine Rule

The Sine and Cosine Rules are used for finding missing sides and angles on non right angled triangles.

Finding a missing side

Step 1: Label your sides and angles

Step 2: Substitute known values into the formula

Step 3: Rearrange the formula to find the missing side

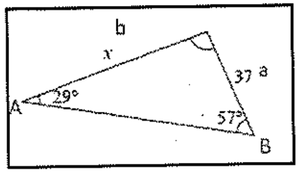
Find the length of side x.

$\frac{37}{\sin 29} = \frac{x}{\sin 57}$

$\frac{37 \times \sin 57}{\sin 29} = x$

$x = 64.004°$

$x = 64°$



Finding a missing angle

$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$

$\frac{SinA}{a} = \frac{SinB}{b}$

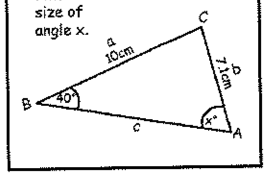
$\frac{SinA}{10} = \frac{Sin40}{7.1}$

$SinA = \frac{Sin40}{7.1} \times 10$

$A = \sin^{-1}\frac{Sin40}{7.1} \times 10$

A = 64.9°

Find the size of angle x.



The formula for the sine rule is

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Cosine Rule

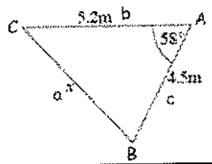
Cosine Rule: Finding a missing side

Can be used to find missing sides or angles. MUST have 2 sides and 1 angle

Formula for missing side

$a^2 = b^2 + c^2 - 2bcCosA$

Find the length of side x.



$x^2 = 5.2^2 + 4.5^2 - (2 \times 5.2 \times 4.5 \times Cos58)$

$x^2 = 5.2^2 + 4.5^2 - (24.8)$

$x^2 = 27.04 + 20.25 - (24.8)$

$x^2 = 47.29 - (24.8)$

$x^2 = 22.49$ (Square root both sides)

$x = 4.74m$

Cosine Rule: Finding a missing angle

Formula for missing angle

$$CosA = \frac{b^2 + c^2 - a^2}{2bc}$$

To find a missing angle… **MUST** have all 3 sides given

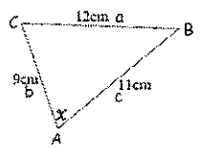
$Cosx = \frac{9^2 + 11^2 - 12^2}{2 \times 9 \times 11}$

$Cosx = \frac{58}{198}$

$x = \cos^{-1}\frac{58}{198}$

$x = 72.97°$

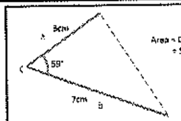
Find the size of angle x.



Area of a triangle

$\text{Area} = \frac{1}{2} a\, b \sin(C)$

Where C is the angle wedged between two sides a and b.

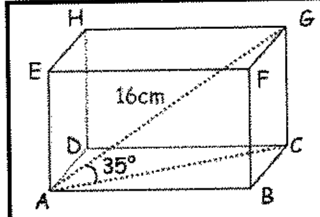


Area = 0.5 × 3 × 7 × sin(59) = 9.00cm²

Trigonometry in 3D

Length AG = 16cm
Angle CAG is 35°

Work out the length of EG.



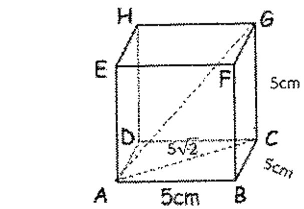
$\cos(CAG) = \frac{a}{h}$

$\cos(35) = \frac{EG}{16}$

$16 \times \cos(35) = EG$

EG = 13.1cm

1. Shown is a cube with side length 5cm.



Calculate angle CAG.

$AC^2 = \sqrt{5^2 + 5^2}$

$AC^2 = \sqrt{50}$

$AC = 5\sqrt{2}$

$\tan(CAG) = \frac{5}{5\sqrt{2}}$

$CAG = \tan^{-1}\frac{5}{5\sqrt{2}}$

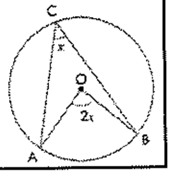
$CAG = 35.3°$

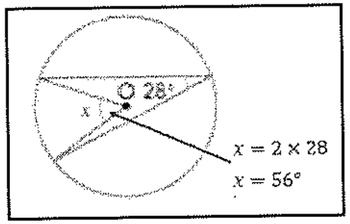
Unit 12: Circle Theorems

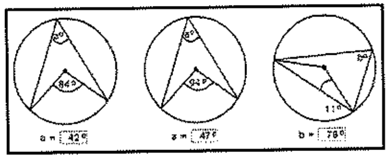
Circle Theorem 1:

"The angle at the centre of a circle is twice the angle at the circumference."

Angle AOB = 2 x ACB

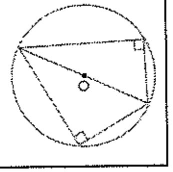


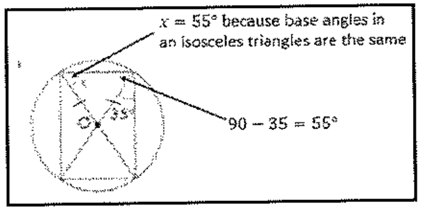


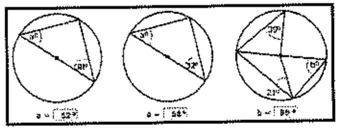


Circle Theorem 2:

"Every angle at the circumference of a semicircle that is subtended by the diameter of the semicircle is a right-angle."



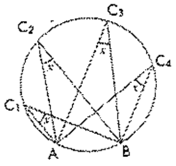


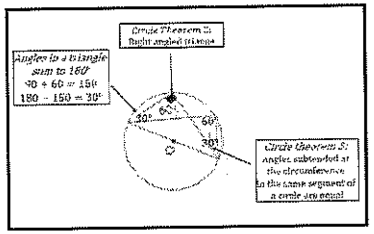


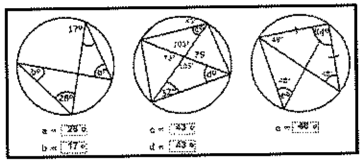
Circle Theorem 3:

"Angles subtended at the circumference in the same segment of a circle are equal."

Points C_1, C_2, C_3 and C_4 on the circumference are subtended by the same arc, AB.



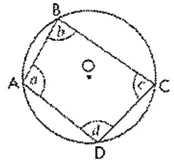


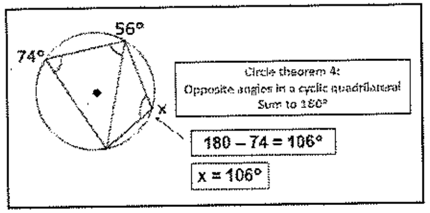


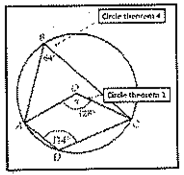
Circle Theorem 4:

"The sum of the opposite angles in a cyclic quadrilateral is 180°."

a + c = 180° and b + d = 180°

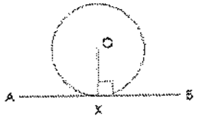


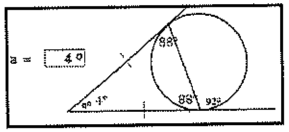




Circle Theorem 5:

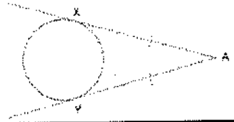
"When a radius meets a tangent, it always makes a 90° angle."





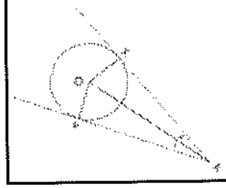
Circle Theorem 6:

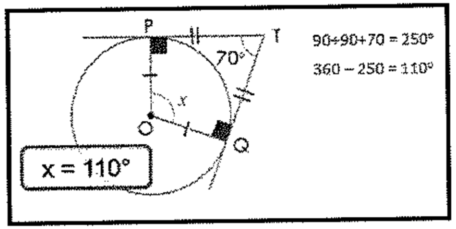
"Tangents to a circle from an external point to the points of contact are equal in length."



Circle Theorem 7:

"The line joining an external point to the centre of the circle bisects the angle between the tangents."

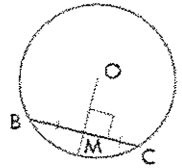


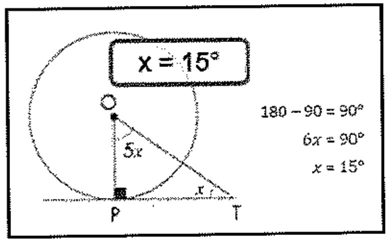


Circle Theorem 8:

"A radius bisects a chord at 90°."

If O is the centre of the circle, angle BMO = 90° and BM = CM.

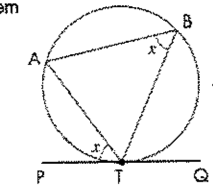


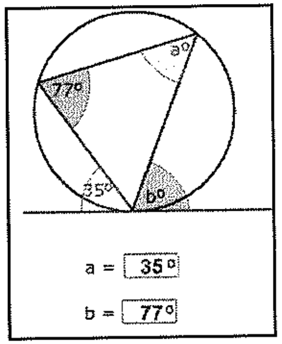


Circle Theorem 9:

The Alternate Segment Theorem

"The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment."





Unit 13: Compound Measures & Proportion

Growth and Decay

Depreciation

I buy a car for £20000.

It depreciates at a rate of 4% *per annum*

What will it be worth after 3 years?

Initial amount= £20000
Depreciation rate = 4%
Multiplier is × 0.96
It depreciated for 3 years

$20000 \times 0.96^3 = £17694$

Compound Interest

Initial amount $\times$ (1 + the rate of interest)^{years}

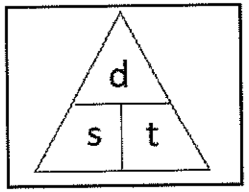
$A(1+r)^n$

You save £2000 in a savings account for 4 years.
The interest rate is 0.6% per annum.

Initial amount= 2000
Interest rate = 0.6%
Multiplier is × 1.006
It is in the bank for 4 years

2000×1.006^4
=£2048.43

Speed, Distance, Time



$$speed = \frac{distance}{time}$$

$$time = \frac{distance}{speed}$$

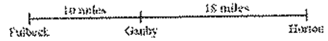
$$distance = speed \times time$$

Use the formulas to convert between compound measures

Speed	Distance	Time
2m/s	10m	5s
5m/s	24m	4.8s
0.2km/s	50km	250s
5km/h	10km	7200s

Compound Measures

The distance from Fulbeck to Ganby is 10 miles.
The distance from Ganby to Horton is 18 miles.



Raksha is going to drive from Fulbeck to Ganby
Then she will drive from Ganby to Horton.

Raksha leaves Fulbeck at 10 00.
She drives from Fulbeck to Ganby at an average speed of 40mph.

Raksha wants to get to Horton at 10 35.

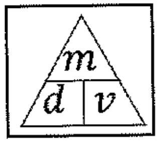
Work out the average speed Raksha must drive at from Ganby to Horton.

Use the rows for speed, distance and time, and the columns for each leg of the journey:

	F → G	G → H
Speed	40 mph	54 mph
Distance	10 miles	18 miles
Time	1/4 hr	1/3 hrs

Density, Mass and Volume

Density gives us a measure of how tightly packed matter is within a space.



$$density = \frac{mass}{volume}$$

$$volume = \frac{mass}{density}$$

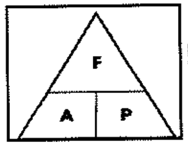
$$mass = density \times volume$$

Again, the unit of g/cm³ allows you to work out the formula for density if you forget.

The density of a rock is 2.3 g/cm³.
Work out the mass of a piece of this rock with a volume of 20 cm³.

Intuitively, if there is 2.3 g for each cm³, then for 20cm³, mass must be
$2.3 \times 20 = 46$ g

Pressure, Force and Area



$$Pressure = \frac{Force}{Area}$$

$$Area = \frac{Force}{Pressure}$$

$$Force = Pressure \times Area$$

What pressure does
a chicken weighing 80N with feet area of 0.1m^2
put on the ground

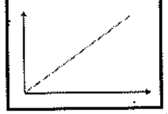
$$\frac{80}{0.1m^2} = 8N/m^2$$

Compound Measures

Direct Proportion

Cost (£) is directly proportional to number of items (x)

$£ \propto x$



Where one quantity directly scales with another (e.g. as one doubles, the other doubles) we say they are directly proportional

Solving Proportion Problems

Steps:

1) Write the proportionality statement
2) Find the equation connecting d and t
3) Substitute the values of d and t to find k
4) Use K and the value given to find the answer

Y is directly proportional to x

When y = 12 x = 4

1) Find the equation linking y to x
2) Find the value of y when x = 6

1) $y \propto x$
$y = kx$
$12 = 4k$
$3 = k$
$y = 3x$

2) $y = 3x$
$y = 3(6)$
$y = 18$

Indirect Proportion

When one quantity increases as another decreases we say they are **indirectly or inversely proportional.**

$$s \propto \frac{1}{t}$$

p is inversely proportional to m

When p = 70 m = 2

1. Find the equation that connects y and m

2. Find p when m = 4

Steps:

1) Write the proportionality statement
2) Find the equation connecting d and t
3) Substitute the values of d and t to find k
4) Use K and the value given to find the answer

1
$p \propto \frac{1}{m}$
$p = \frac{k}{m}$
$70 = \frac{k}{2}$
$k = 140$
$p = \frac{140}{m}$

2
$p = \frac{140}{4}$
$p = 35$

Proportion

Unit 14: Further Algebra

Algebraic Fractions

When we multiply fractions we multiply the numerators together and the denominators together

If the fraction shares common factors then we can cross cancel by dividing the factors out before we multiply

$\frac{7}{14} \times \frac{35}{7} = \frac{1}{1} \times \frac{5}{1} = 5$

$\frac{a^2}{2} \times \frac{4}{a} = \frac{a}{1} \times \frac{2}{1} = \frac{2a}{1} = 2a$

$\frac{3(c+2)}{8c} \times \frac{2c(c-1)}{c+2} = \frac{3}{4} \times \frac{c-1}{1} = \frac{3(c-1)}{4}$

When we divide fractions we find the reciprocal of the second fraction and then multiply the fractions together.

$\frac{6x}{2y} \div \frac{4y}{5} = \frac{6x}{2y} \times \frac{5}{4y} = \frac{15}{4y^2}$

When we add or subtract with fractions we need to have the same denominator in each fraction.

To do this we need to find the lowest common denominator

$\frac{x}{3} + \frac{2x+1}{2} = \frac{2x + 3(2x+1)}{3 \times 2}$ LCM = 6

$= \frac{2x + 6x + 3}{6}$

$= \frac{8x+3}{6}$

Step 1: Find the lowest common denominator

Step 2: Cross multiply

Step 3: Simplify

Solve $\frac{x+5}{7} = 5$

$\times 7 \quad \times 7$

$x + 5 = 35$

$-5 \quad -5$

$x = 30$

Solve the equation $\frac{x+4}{2} = \frac{x+10}{3}$ ($\times 3$, $\times 2$)

$3(x+4) = 2(x+10)$

$3x + 12 = 2x + 20$

$3x = 2x + 8$

$x = 8$

Solve

$\frac{2x+1}{3} + \frac{x-5}{2} = 4$ LCD = 6

$\frac{2(2x+1) + 3(x-5)}{3 \times 2} = 4$

$\frac{4x + 4 + 3x - 15}{6} = 4$

$\frac{7x - 11}{6} = 4$

$\times 6 \quad \times 6$

$7x - 11 = 24$

$+11 \quad +11$

$7x = 35$

$\div 7 \quad \div 7$

$x = 5$

Step 1: Find the lowest common denominator

Step 2: Cross Multiply

Step 3: Simplify the numerator

Step 4: Solve

Changing the subject

1. Circle the subject that we want to isolate
2. Identify what needs to be eliminated
3. Inverse the operation, and apply to both sides

When we have two terms on one side:

1) Factorise out the subject
2) Circle the subject that we want to isolate
3) Identify what needs to be eliminated
4) Inverse the operation, and apply both sides

Make x the subject of:

$ax + b = cx + d$

$ax - cx = d - b$

$x(a - c) = d - b$

$x = \frac{d-b}{a-c}$

Make c the subject

$cm + ca = z$

$c(m + a) = z$

$\div m + a \quad \div m + a$

$c = \frac{z}{m+a}$

Make x the subject of:

$q = \frac{2x+1}{2x-1}$

$q(2x-1) = 2x + 1$

$2xq - q = 2x + 1$

$2xq - 2x = 1 + q$

$x(2q - 2) = 1 + q$

$x = \frac{1+q}{2q-2}$

Surds

Simplifying Surds

Simplify

$\sqrt{60}$

$\sqrt{4 \times 15}$

$\sqrt{4}\sqrt{15}$

$2\sqrt{15}$

1) Find two factors of the number (one of them needs to be a square number)

2) Chop the surd in half

3) Simplify

$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$

Simplify $\sqrt{2} + \sqrt{8}$

When the numbers under the square root are different we have to simplify first

$= \sqrt{2} + \sqrt{4}\sqrt{2}$

$= \sqrt{2} + 2\sqrt{2}$

$= 3\sqrt{2}$

Multiplying Surds

$\sqrt{5} \times \sqrt{6} = \sqrt{30}$

$(2 + \sqrt{3})(2 + \sqrt{3})$

x	2	$\sqrt{3}$
2	4	$2\sqrt{3}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{9}$

$= 4 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{9}$

$= 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$

Rationalising the denominator

Rationalise the denominator for $\frac{3}{\sqrt{5}}$

$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

Rationalise the denominator for $\frac{2}{\sqrt{3}+1}$

$= \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2\sqrt{3}-2}{3 - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{2\sqrt{3}-2}{2}$

$= \sqrt{3} - 1$

Algebraic Proof

SHOW...

"SHOW THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

1 + 2 = 3 ✓

2 + 3 = 5 ✓

5 + 6 = 11 ✓

101 + 102 = 203 ✓

We have SHOWN that this works. We have not proved it works.

There could be, at some point, two consecutive numbers in fact give us an EVEN number

PROVE... ← Algebra is often used to establish the truth (to prove something)

"PROVE THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

We can give any number, any letter. In this case, lets pick the letter "n"

Therefore the number directly after n is "n+1"

"SUM OF TWO CONSECUTIVE NUMBERS."

n + n + 1 = 2n + 1

"2n" is the nth term for the multiples of 2.

2n + 1 is one more than the multiples of 2, the odd numbers.

This PROVES that the statement is always true for any value of n

Identity:

An identity in maths is represented by the ≡ symbol

An identity is an equation which is always true no matter which values are chosen

$(x+1)^2 \equiv x^2 + 2x + 1$

"Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 4 for all positive integer values of n"

*Before you begin, highlight the key words and information from the question.

Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 4 for all positive integer values of n"

STEP 1: EXPAND

$(3n+1)^2 - (3n-1)^2$

$= (3n+1)(3n+1) - (3n-1)(3n-1)$

$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

STEP 2: SIMPLIFY

$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

$= 12n$

STEP 3: FACTORISE

$12n = 4 \times 3n$

STEP 3: JUSTIFY

$4 \times 3n \rightarrow$ always divisible by 4 and hence a multiple of 4

Prove that $(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n

Expand

$= (n+1)(n+1) - (n-1)(n-1) + 1$

$= (n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$

Simplify $= 4n + 1$

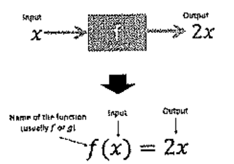
Justify $4 \times n$ is always even

Any even number add 1 is odd

Functions

A function is something which provides a rule on how to map inputs to outputs.

From primary school you might have seen this as a 'function machine'.



If $p = 3x$ and $x = \frac{y}{2}$

Write p in terms of y

$p = 3x$

$p = 3\left(\frac{y}{2}\right)$

$p = \frac{3y}{2}$

You are given that $f(x) = 3x + 1$

Find $f(1)$

This means we need to substitute x for 1

Find $f(1) = 3(1) + 1 = 3 + 1 = 4$

$f(x) = x^2 + 2$

If $f(a) = 38$, what is a?

$x^2 + 2 = 38$

$x^2 = 36$

$x = \pm 6$

If $f(x) = x + 1$ what is:

$f(x+1) = (x+1) + 1 = x + 2$

$f(x-1) = (x-1) + 1 = x$

$f(x^2) = x^2 + 1$

$f(x)^2 = (x+1)^2 = x^2 + 2x + 1$

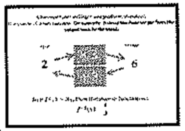
$f(2x) = 2x + 1$

$2f(x) = 2(x+1) = 2x + 2$

Anything inside the bracket applies just to the x

Anything outside the bracket applies just to the whole function

Inverse Functions



$f(x) = 2x + 1$

find $f^{-1}(x)$

$y = 2x + 1$

$y - 1 = 2x$

$x = \frac{y-1}{2}$ $\quad f^{-1}(x) = \frac{x-1}{2}$

STEP 1: Write the output $f(x)$ as y

STEP 2: Get the input in terms of the output (make x the subject).

STEP 3: Swap y back for x and x back for $f^{-1}(x)$.

Composite Functions

Given $f(x) = 2x$
$g(x) = x + 1$
find $fg(2)$

$g(2) = 2 + 1 = 3$

$f(3) = 2 \times 3 = 6$

Given $f(x) = 2x$
$g(x) = x + 1$
find $fg(1)$

$g(1) = 1 + 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(x) = x + 1$
$g(x) = 2x$

Find $gf(x)$

$f(x) = x + 1$

$g(x+1) = 2(x+1) = 2x + 2$

$gf(x) = 2x + 2$

Unit 15: Further Statistics

Sampling

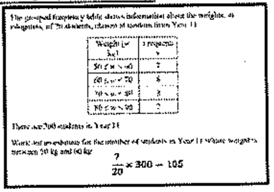
When you are investigating a hypothesis, the population is the whole group that you are interested in.

A population is everything or everybody that could possibly be involved in an investigation

A population may divide into groups such as age range These groups are called strata.

In a stratified sample, the number of people taken from each group is proportional to the group size.

Stratified Sampling



Capture Recapture

Capture-Recapture is a technique that can be used to estimate the total population.

$$\frac{M}{N} = \frac{R}{T}$$

M = total marked
N = total population
R = number "recaptured"
T = total capture on 2nd visit

Clive wants to estimate the number of bees in a beehive.
Clive catches 50 bees from the beehive.

He marks each bee with a dye
He then lets the bees go.

The next day, Clive catches 40 bees from the beehive.
8 of these bees have been marked with the dye.

(i) Work out an estimate for the number of bees in the beehive.

M = 50
T = 40
R = 8
N = ?

$$\frac{50}{n} = \frac{8}{40}$$

$$50 = \frac{8n}{40}$$

$$2000 = 8n$$

$$250 = n$$

Cumulative Frequency

A Cumulative Frequency table shows a running total of the frequencies

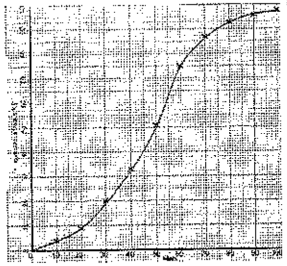
Mark	Frequency	Cumulative Frequency
1-10	3	3
11-20	6	3 + 6 = 9
21-30	11	9 + 11 = 20
31-40	13	20 + 13 = 33
41-50	18	33 + 18 = 51

A cumulative frequency table shows how many data values Are less than or equal to the upper class boundary of each data class

The upper class boundary is the highest possible value in each class.

Drawing a Cumulative Frequency Graph

Mark	Frequency	Cumulative Frequency
1-10	3	3
11-20	6	9
21-30	11	20
31-40	13	33
41-50	18	51
51-60	24	75
61-70	12	87
71-80	6	93
81-90	3	96
91-100	2	98



Steps:

1) Start from 0
2) Plot using end points
3) Join using a smooth curve

Averages

1 1 3 5 7 9 11 14 16 19 19 20 21

n = 13

$\text{Median} = \frac{13+1}{2} = \frac{14}{2} = 7th\ value = 11$

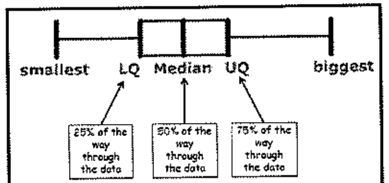
$\text{LQ} = \frac{13+1}{4} = \frac{14}{4} = 3.5th\ value = 4$

$\text{UQ} = 3.5 \times 3 = 10.5th\ value = 19$

Median formula $= \frac{n+1}{2}$

Lower Quartile formula $= \frac{n+1}{4}$

Upper Quartile formula $= LQ \times 3$

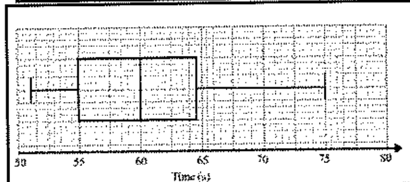


Drawing a box plot

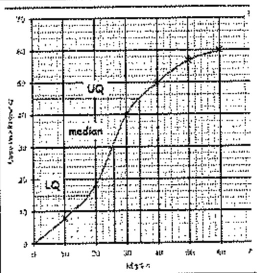
The times, in seconds, of 15 students running a race are recorded below.

52 54 54 55 56 58 59 60 60 63 65 64 67 70 75

Draw a box plot for this information.



Box Plots + CF Diagrams



Median – Middle value found at total frequency ÷ 2

Lower quartile – Value which is a quarter of the way through the data found at total frequency ÷ 4

Upper quartile – Value which is three quarters of the way through the data Found at total frequency ÷ 4 × 3

Interquartile range = Upper quartile – Lower quartile

Median = 26

Lower Quartile = 18

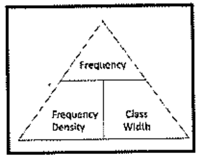
Upper Quartile = 34

IQR= 34 – 18 =16

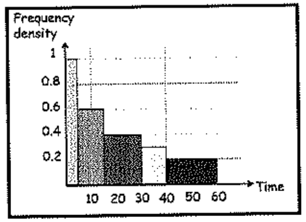
Histograms

Drawing a histogram

Time, t, in minutes	Frequency density	Frequency
0 ≤ t < 5	1	5
5 ≤ t < 15	0.6	6
15 ≤ t < 30	0.4	6
30 ≤ t < 40	0.3	3
40 ≤ t < 60	0.2	4



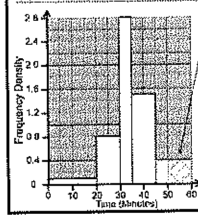
This is the formula triangle for a histogram



Interpreting a Histogram

The histogram shows the times a sample of students spent on the internet one evening. (a) Copy and complete the frequency table.
(b) Estimate how many students spent longer than 50 minutes on the internet.

Time, t, minutes	0 ≤ t < 20	20 ≤ t < 30	30 ≤ t < 35	35 ≤ t < 45	45 ≤ t < 60
Frequency	0.1 × 20 = 2	0.8 × 10 = 8	2.8 × 5 = 14	1.5 × 10 = 15	0.4 × 15 = 6



50 Minutes or more

The area of the histogram represents area.

Area 50 ≤ t < 60

= 0.4 × 10 = 4

4 students spent longer than 50 minutes on the internet.

Unit 16: Probability

Probability Events

Single Events

P(event) = $\frac{\text{outcomes matching event}}{\text{total outcomes}}$

Probabilities add up to 1

If P(event) = 0.4

The P(not event) = 1-0.4 = 0.6

Combined Events

Listing Outcomes

I flip two non biased coins.

List systematically all the outcomes

HH
HT
TH
TT

There are 4 outcomes if we flip two non biased coins

Sample Space Diagrams

Sample space diagrams are a way of showing multiple outcomes in one diagram.

After throwing 2 fair die and adding.

2nd Die (across), 1st Die (down)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

What is the probability that after throwing two dice you get a prime number?

There are 15 prime numbers, and 36 outcomes all together,

So the probability is $\frac{15}{36}$

Finding missing probabilities from a table

As probabilities sum to 1, you must add together all the known probabilities and then subtract from 1

Red	Blue	Yellow	Green
0.1	x	0.4	0.2

$0.1 + 0.4 + 0.2 = 0.7$

$1 - 0.7 = 0.3$

Expected and Relative frequency

Expected Frequency

Expected frequency of an event = probability of event × number of trials

This is also known as theoretical probability

I throw an unbiased dice 60 times.

How many times would you expect to roll a 6?

Probability of the event is $\frac{1}{6}$

Number of trials = 60

$$\frac{1}{6} \times 60 = 10$$

Relative Frequency

$$relative\ frequency = \frac{number\ of\ successes}{number\ of\ trials}$$

This is also known as experimental probability

Score	Frequency	Relative Frequency
1	12	= 12 ÷ 50 = 0.24
2	25	= 25 ÷ 50 = 0.5
3	9	= 9 ÷ 50 = 0.18
4	4	= 4 ÷ 50 = 0.12

Total of 50 spins

Relative frequency is called experimental probability because it has already happened.

Expected frequency is called theoretical probability because it has not yet happened.

Tree Diagrams

Independent Events

If A and B are independent events, then the outcome of one doesn't affect the other. Then:

P(A and B) = P(A) × P(B)

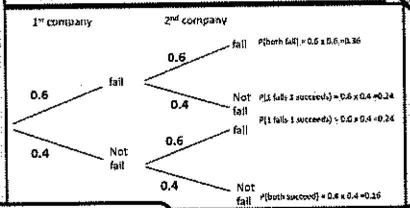
Mutually Exclusive Events

If A and B are mutually exclusive events, they can't happen at the same time. Then:

P(A or B) = P(A) + P(B)

Tree Diagrams: Independent Events

The probability that a new company will fail in its first 5 years is 0.6. Two companies are chosen at random.

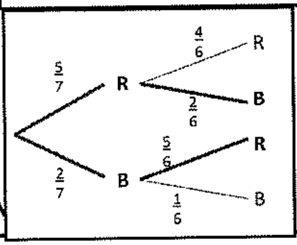


You always multiply along the branches in a tree diagram

Conditional Probability

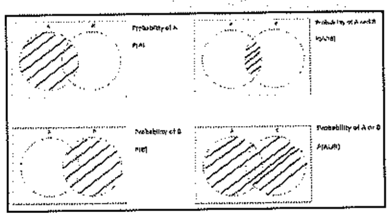
Conditional Probability is the probability of an event (A), given that another (B) has already occurred.

Given there's 5 red balls and 2 blue balls. If I taken two balls without replacement, what will the probability tree look like?



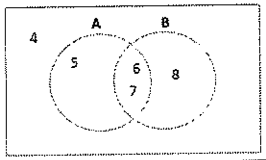
Venn Diagrams

Sets and Venn Diagrams



This symbol means all the numbers in the set

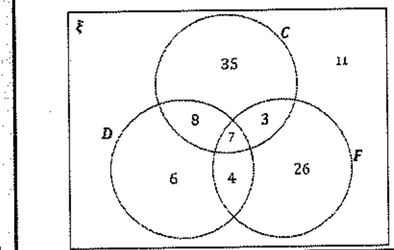
$\xi = \{4,5,6,7,8\}$, $A = \{5,6,7\}$
$B = \{6,7,8\}$
Construct a Venn Diagram to show these sets.



Venn Diagrams with frequencies

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Fill in this Venn Diagram, and hence answer the following questions:
a) *P(owns dog only)*
b) *P(does not own tropical fish)*
c) *P(does not own dogs, cats, or tropical fish)*



a) $\frac{6}{100}$

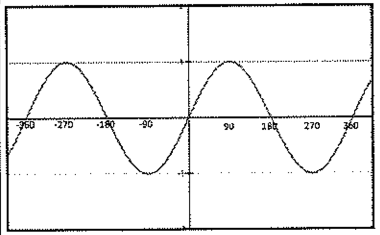
b) $\frac{60}{100}$

c) $\frac{11}{100}$

Unit 17: Trigonometric Graphs

The graphs of sine, cos and Tan

Sine Graph



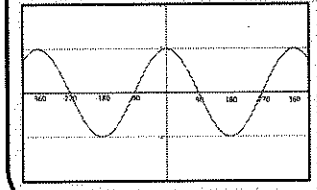
The Sine Graph repeats every 360°

It's largest value is 1

It's smallest value is -1

Each graph is shown between the range of –360 degrees and + 360 degrees. The range can be changed/restricted so make sure you read the range carefully in a question.

Cos Graph



The Cosine Graph repeats every 360°

It's largest value is 1

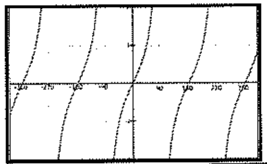
It's smallest value is -1

Tan Graph

The Tan Graph repeats every 180°

There are asymptotes (the dashed lines) that the tan graph never touches.

It doesn't have a maximum or minimum point, it goes between negative infinity and positive infinity.



Solving Equations with Trig Graphs

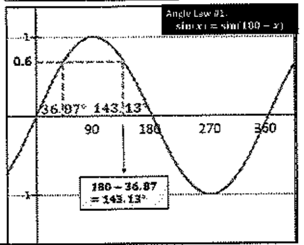
Solve sin(x)=0.6 in the range 0≤x<360

1) Draw a straight horizontal line at 0.6 (you can see we will have 2 solutions)

2) Substitute 0.6 into the inverse trig function into your calculator

3) Round to a suitable degree of accuracy: 36.87°

4) Subtract 36.87° from 180° to find the second solution.



$x = \sin^{-1}(0.6) = 36.87°, 143.13°$

1) Rearrange first to eliminate the 3

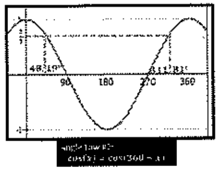
2) Draw a horizontal line at $\frac{2}{3}$

3) Substitute $\frac{2}{3}$ into the inverse cos function

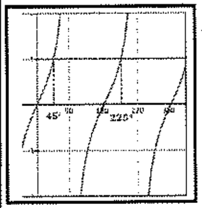
4) Round the answer to a suitable degree of accuracy

5) Find all solutions

Solve $3\cos(x) = 2$ in the range $0 \leq x < 360$



$x = cos^{-1}\left(\frac{2}{3}\right) = 48.19°, 311.81°$



Solve $\tan(x) = 1$ in the range $0 \leq x < 360$

$x = \tan^{-1}(1) = 45°$

$45° + 180° = 225°$

1) Draw a horizontal line at 1

2) Substitute **1** into the inverse tan function

3) Find all solutions

Steps

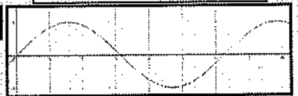
1) Sketch the trig graph within the specified range.
2) Draw a straight horizontal line at your given value notice how many times it crosses the graph (this is how many answers there are)
3) Substitute the value given into the **inverse trig function** into your calculator.
4) Round to a suitable degree of accuracy
5) This gives you **one** answer
6) Use your answer to find other solutions of the equation.

Transformations of Graphs

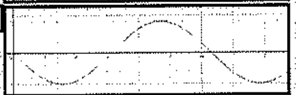
When we have a negative change inside the bracket

$f(-x)$ *is a reflection of the graph in the y − axis*

$y = \sin x$



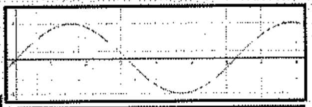
$y = \sin(-x)$



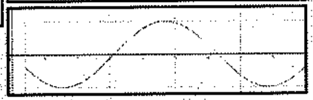
When we have a negative change outside the bracket

$-f(x)$ *is a reflection of the graph in the x − axis*

$y = \sin x$



$y = -\sin x$



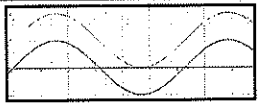
When we add or subtract outside the bracket it translates the graph in y

$f(x) + 1$ *translates the graph by* $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$f(x) - 1$ *translates the graph by* $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$y = \sin x$

$y = \sin x + 1$



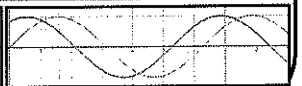
When we add or subtract inside the bracket it translates the graph in x (in the opposite direction)

$f(x+1)$ *translates the graph by* $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$f(x-1)$ *translates the graph by* $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$y = \sin x$

$y = \sin(x+1)$

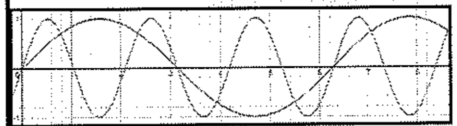


When we have a change inside the bracket

$f(ax)$ *squashes on the x axis by a factor of a*

$y = \sin x$

$y = \sin 3x$



f(3x) → Squash on x-axis by factor of 3

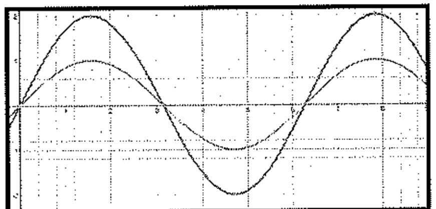
When we have a change outside the bracket

$af(x)$ *stretched on the y − axis by a factor of a*

2f(x) → Stretch on y-axis by factor of *2*

$y = \sin x$

$y = 2\sin x$



Unit 18: Vectors

Vector Basics

What is a vector?

A **vector** describes *direction* and *length*

The magnitude of a vector is its size

$\begin{pmatrix} x \\ y \end{pmatrix}$

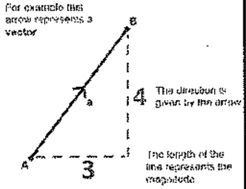
X = number of moves to the **right** or **left**

Y= number of moves **up** or **down**

This vector can be written in 3 ways

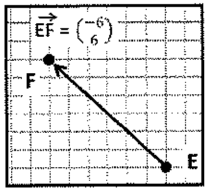
a $\bar{a}$ $\overrightarrow{AB}$

For example this arrow represents a vector

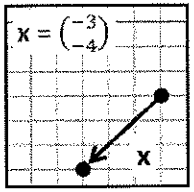


$a = \binom{3}{4}$ $\bar{a} = \binom{3}{4}$ $\overrightarrow{AB} = \binom{3}{4}$

$\overrightarrow{EF} = \binom{-6}{6}$



$x = \binom{-3}{-4}$

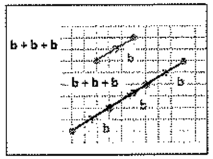


Vector Arithmetic

What's another way of saying b + b + b?

3b

Scalar ← → Vector



If we add two or more vectors together we get a resultant vector

A resultant vector is the vector sum of two or more vectors

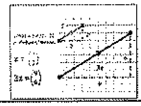
$\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

Multiplying a Vector by a Scalar

$z = \binom{3}{2}$

$3z = \binom{9}{6}$



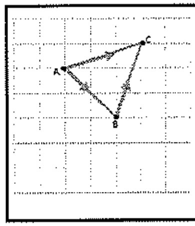
To multiply a Vector by a Scalar. Write the Vector as a Column Vector then multiply each entry in the Column Vector by the Scalar

$3z = \binom{3}{2}\begin{matrix}\times 3\\ \times 3\end{matrix} = \binom{9}{6}$

We can multiply a vector by a scalar

A scalar is a quantity that has size but no direction

Vectors that have been multiplied by a scalar are **parallel**



$\overrightarrow{AB} = \binom{2}{-2}$

$\overrightarrow{BC} = \binom{1}{3}$

$\overrightarrow{AB} + \overrightarrow{BC} = \binom{3}{1}$

$\overrightarrow{BC} - \overrightarrow{AB} = \binom{-1}{5}$

Midpoints and Ratio

Midpoints of Vectors

3. *P* is the point (1,5), *Q* is the point (9,3)

a) Write down the vector $\overrightarrow{PQ}$ $\binom{8}{-2}$

Write your answer as a column vector

M is the midpoint of PQ

$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} = \binom{4}{-1}$

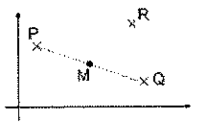
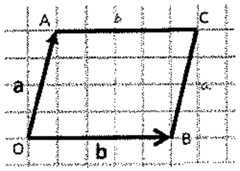


Diagram **NOT** accurately drawn

Vectors in quadrilaterals

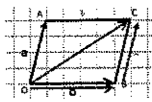


OACB is a parallelogram

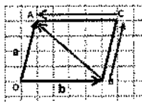
$\overrightarrow{OA}$ = a and $\overrightarrow{OB}$ = b

Find i) $\overrightarrow{OC}$ ii) $\overrightarrow{BA}$ iii) $\overrightarrow{CA}$

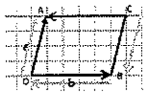
In terms of a and b



$\overrightarrow{OC} = b + a$



$\overrightarrow{BA} = a - b$



$\overrightarrow{CA} = -b$

$\overrightarrow{CA} = -a - b + a$

$\overrightarrow{CA} = -b$

Vectors with ratio

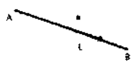
$\overline{AL} : \overline{LB} = 2 : 1$

What is:

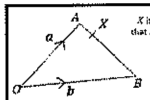
$\overrightarrow{AB} = a$

$\overrightarrow{AL} = \frac{2}{3}a$

$\overrightarrow{LB} = \frac{1}{3}a$



There are 3 parts to the ratio So we are dealing with thirds



X is a point such that *AX*:*XB* = 1:4

a. $\overrightarrow{AB} = -a + b$

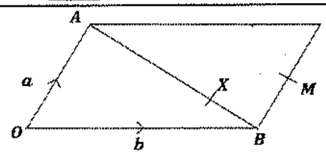
b. $\overrightarrow{AX} = -\frac{1}{5}a + \frac{1}{5}b$

c. $\overrightarrow{OX} = \frac{4}{5}a + \frac{1}{5}b$

d. $\overrightarrow{BX} = \frac{4}{5}a - \frac{4}{5}b$

Vector Proof

How to show two vectors are parallel



X is a point on *AB* such that *AX*:*XB* = 3:1. *M* is the midpoint of *BC*. Show that $\overline{XM}$ is parallel to $\overline{OC}$.

$\overrightarrow{OC} = a + b$

$\overrightarrow{XM} = \frac{1}{4}(-a+b) + \frac{1}{2}a = \frac{1}{4}a + \frac{1}{4}b$

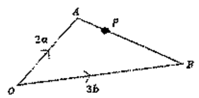
$= \frac{1}{4}(a+b)$

$\overrightarrow{XM}$ **is a multiple of** $\overrightarrow{OC}$ ∴ parallel.

For any proof question always find the vectors involved first, in this case $\overrightarrow{XM}$ and $\overrightarrow{OC}$.

The key is to factor out a scalar such that we see the same vector.

The magic words here are "*is a multiple of*".



a) Find $\overrightarrow{AB}$ in terms of *a* and *b*.

$-2a + 3b$

b) *P* is the point on *AB* such that *AP*:*PB* = 2:3.

Show that $\overrightarrow{OP}$ is parallel to the vector *a* + *b*.

M1 for $2a + \frac{2}{5}(3b - 2a)$ OR $3b + \frac{3}{5}(2a - 3b)$

A1 for $\frac{6}{5}a + \frac{6}{5}b$ oe

A1 for $\frac{6}{5}(a + b)$ is parallel to a + b oe

Collinear Points

Points A, B and C form a straight line if: $\overrightarrow{AB}$ **and** $\overrightarrow{BC}$ **are parallel (and B is a common point).**

Alternatively, we could show $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. This tends to be easier.