Highsted Grammar School

GCSE to A Level Mathematics

Transition Booklet

"We all use maths every day. To predict weather…to tell time…to handle money. Maths is more than formulas and equations. It's logic; it's rationality. It's using your mind to solve the biggest mysteries we know."

Name: ______________________

Introduction to Mathematics A-Level at

Highsted Grammar School.

Thank you for choosing to study Mathematics in the sixth form at Highsted Grammar School. You will sit two modules in Pure Mathematics as well as a combined module of Statistics and Mechanics. If you have chosen to study Further Mathematics as well as Maths then, in year 12, you will additionally study modules in Further Pure 1 and Further Pure 2. The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have prepared this booklet.

It is vitally important that you spend time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September. You will have met all the topics before at GCSE. Read through the introduction to each chapter, making sure that you understand the concepts; then tackle the exercise enough to ensure you understand the topic thoroughly. The answers are given at the back of the booklet.

All of the topics covered in this booklet are skills that are essential to ensure a smooth transition from GCSE Mathematics and A-Level Mathematics. If you do not have a solid understanding of these topics then you will find that you may struggle to grasp some concepts at A-Level Mathematics.

We hope that you will use this introduction to give you a good start to you're A-Level work and that it will help you enjoy and benefit from the course more. You must bring this booklet with you to your first Maths lesson in September.

Mrs L Allen

Assistant Headteacher

Leader of Mathematics

CONTENTS

Chapter 1: FACTORISING

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$

The method is:

<u>Step 1</u>: Form two brackets $(x \;\ldots\;)(x \;\ldots\;)$

<u>Step 2</u>: Find two numbers that multiply to give *c* and add to make *b*. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.

Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$

The method is:

<u>Step 1</u>: Find two numbers that multiply together to make *ac* and add to make *b*.

<u>Step 2</u>: Split up the *bx* term using the numbers found in step 1.

<u>Step 3</u>: Factorise the front and back pair of expressions as fully as possible.

<u>Step 4</u>: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make 6 × -12 = -72 and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x}\; \underbrace{+ 9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Exercise A

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2 y$

14) $8(x+1)^2 - 2(x+1) - 10$

Chapter 2: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced. Below is a more complex example where the letter to be made a subject is on both sides of the equation.

Example 1: Make *W* the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by 2*b*: $2bT - 2bW = Wa$

Add 2*bW* to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by *a* + 2*b*: $W = \frac{2bT}{a + 2b}$

Exercise A

Make *t* the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

Exercise B

Make *x* the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x + 3}{5x - 2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

Chapter 3: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:

* factorising
* the quadratic formula
* completing the square

Note that not all quadratic equations can be solved by factorising. The quadratic formula and completing the square can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Solution:

Factorise $(x - 1)(x - 2) = 0$

Either $(x - 1) = 0$ or $(x - 2) = 0$

So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$

We can then tell that $a = 2$, $b = 3$ and $c = -12$.

Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

Method 3: Completing the Square

Example 3: Solve the equation $x^2 + 6x - 11 = 0$

Solution: First we half the coefficient of the x term. Then put that in a squared bracket. Copy down the term without an x and finally always subtract the square of the number in the bracket.

$$(x + 3)^2 - 4 - 3^2 = 0$$
$$(x + 3)^2 - 11 - 9 = 0$$
$$(x + 3)^2 - 20 = 0$$

Now we solve this:

$(x + 3)^2 = 20$ add 20 to both sides

$x + 3 = 20$ square root both sides

$x = -3 \pm \sqrt{20}$ subtract 3 from both sides

Now we can leave it in simplified surd form

$$x = -3 \pm 2\sqrt{5}$$

EXERCISE A

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

5) Use completing the square to solve the following equations. Give your answer in exact (surd) form.

a). $x^2 - 8x - 5 = 0$

b). $x^2 + 6x - 15 = 0$

c). $x^2 + 8x + 1 = 0$

d). $x^2 - 10x - 1 = 0$

Chapter 4: Sketching Curves and using the Discriminant

We need to be able to sketch graphs of quadratic functions using the discriminant and other facts about roots.

The Discriminant

The discriminant is found in the quadratic formula, it is the part of the formula that dictates how many solutions a quadratic equation has.

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant is the value under the surd, hence we have $b^2 - 4ac$

Since we are both adding and subtracting to generate our values of x, then the value of this surd plays a part in the values of the solution.

If $b^2 - 4ac > 0$, hence positive value, then we can root it and we will add and subtract some value which generates two different solutions to the quadratic equation.
If $b^2 - 4ac = 0$, then we root zero and obtain zero, so we add and subtract zero, giving the same value for both solutions.
If $b^2 - 4ac < 0$, hence negative, then we cannot root this value in the real numbers. This means we do not obtain any solutions.

So we can generalise this concept to:

$$b^2 - 4ac \begin{cases} > 0, \text{ there are two unique solutions} \\ = 0, \text{ there is one repeated solution} \\ < 0, \text{ there are no solutions} \end{cases}$$

Example 1:

How many solutions does the quadratic $3x^2 - 5 = 12x$ have?

Solution:

First rearrange to the form $ax^2 + bx + c = 0$
$3x^2 - 12x - 5 = 0$
Then calculate $b^2 - 4ac$
$(-12)^2 - 4(3)(-5) = 144 + 60 = 204$. Since $b^2 - 4ac$ is positive, the quadratic has two unique solutions.

2. Given that $2x^2 - kx + 4$ has one distinct solution, find the value of k.
Here we will let a = 2, b = -k, c = 4, and since there is only one solution then we know $b^2 - 4ac = 0$
Hence,
$(-k)^2 - 4(2)(4) = 0$
$k^2 - 32 = 0$
$k = \pm 4\sqrt{2}$

Exercise A

1. How many times do the following curves cross the x axis?

a). $y = x^2 + 2x + 5$

b). $y = 2x^2 + 12x + 18$

c). $y = 3x^2 - 4x - 7 = 0$

d). $y = 2x^2 - 8x - 19$

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2. Find the values of k for which the equation $y = kx^2 + 8x + k$ has equal roots.

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3. The equation $y = 2x^2 - 3x + 3k$ has one real solution. Find the values of k.

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4. Show that the equation $y = 3x^2 - 2x + 5 = 0$ has no real solutions.

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5. The equation $x^2 - (5 + m)x + 4 = 0$, where m is a constant, has equal roots. Find the values of m.

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6. The equation $x^2 + kx + k = 0$, where k is a constant, has no real roots.

a). Write down the discriminant of $x^2 + kx + k$ in terms of k.

b). Hence find the set of values k can take.

Chapter 5 Simultaneous Equations

Solving simultaneous equations is finding a solution for two variables that satisfies both equations. When we are solving these problems, we are finding where a straight line meets a curve. This should tell us that we expect to get zero (line doesn't meet curve), one (line is tangent to curve) or two (line cuts curve) solutions but no more.

We will use the method of substitution, and will always rearrange the linear equation to create a subject and substitute into the non-linear equation.

Example:
Solve the following equations:

$$y - x = 2$$
$$x^2 + y^2 = 20$$

Solution:
First rearrange the first equation into the form y = … or x = …

$$y = x + 2$$

Now substitute into the second equation, giving

$$x^2 + (x + 2)^2 = 20$$

Expanding brackets and rearranging into a quadratic gives

$$2x^2 + 4x - 16 = 0$$

Which can be simplified to

$$x^2 + 2x - 8 = 0$$

Hence $x = 2$ or $x = -4$, substituting into the first equation gives $y = 4$ or $y = -2$

So our solutions are $x = 2$, $y = 4$ and $x = -4$, $y = -2$

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1. Solve the following simultaneous equations.

a) $y = 3x$, $y = x^2 - 4$

b). $x - y = 4$, $3x^2 + 4xy + y^2 = 0$

c). $2x - y = 5$, $x^2 - xy + y^2 = 7$

d). $y = 3x - 2$, $2x^2 - xy + y^2 = 2$

e). $xy = 12$, $x^2 + y^2 = 40$

f). $4x + 3y = 3$, $20x^2 + 9y^2 = 6$

2. Show that the elimination of y from the simultaneous equations

$xy - y^2 = -3$ and $2x - y = 1$ produces the equation

$$2x^2 - 3x - 2 = 0.$$

Solve this equation and find the pairs (x, y) for which the simultaneous equations are satisfied.

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3. The equation $x^2 + y^2 = 8$ represents a circle of centre (0, 0) and radius $\sqrt{8}$, The line $y = 2x + 2$ crosses the circle at two points. Find the coordinates of the two points of intersection.

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4. Show algebraically that the line $y = 3 - 2x$, crosses the curve $y = x^2$ at two points. Find the coordinates of the points of intersection.

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5. Find the coordinates of the point where the line $y = 3x - 10$ touches the line $x^2 + y^2 = 10$.

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6. Show algebraically that the line $y = x + 4$ is a tangent to the circle $x^2 + y^2 = 8$, stating the coordinates of the point where the line touches the circle.

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7. Show algebraically that the simultaneous equations $x^2 + y^2 = 1$ and $y = x + 5$ have no solutions.

Chapter 6: INDICES

The Rules of Indices are:

1)	$a^m \times a^n = a^{m+n}$	e.g.	$3^4 \times 3^5 = 3^9$		
2)	$a^m \div a^n = a^{m-n}$	e.g.	$3^8 \times 3^6 = 3^2$		
3)	$(a^m)^n = a^{mn}$	e.g.	$\left(3^2\right)^5 = 3^{10}$		
4)	$a^0 = 1$	e.g.	$5^0 = 1$	$\left(\frac{3}{4}\right)^0 = 1$	$\left(-5.2304\right)^0 = 1$
5)	$a^{-n} = \frac{1}{a^n}$	e.g.	$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$		
6)	$a^{1/n} = \sqrt[n]{a}$	e.g.	$8^{1/3} = \sqrt[3]{8} = 2$		
7)	$a^{m/n} = \left(a^{1/n}\right)^m$	e.g.	$4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$		

Exercise A

Simplify the following:

1) $b \times 5b^5$ =

2) $3c^2 \times 2c^5$ =

3) $b^2c \times bc^3$ =

4) $2n^6 \times (-6n^2)$ =

5) $8n^8 \div 2n^3$ =

6) $d^{11} \div d^9$ =

7) $\left(a^3\right)^2$ =

Exercise B:

1) $4^{1/2}$ =

2) $27^{1/3}$ =

3) $\left(\frac{1}{9}\right)^{1/2}$ =

4) 5^{-2} =

5) 18^0 =

6) 7^{-1} =

7) $27^{2/3}$ =

8) $\left(\frac{2}{3}\right)^{-2}$ =

9) $8^{-2/3} =$

10) $(0.04)^{1/2} =$

11) $\left(\frac{8}{27}\right)^{2/3} =$

12) $\left(\frac{1}{16}\right)^{-3/2} =$

13) $2a^{1/2} \times 3a^{5/2} =$

14) $x^3 \times x^{-2} =$

15) $\left(x^2 y^4\right)^{1/2} =$

Chapter 7: SURDS

The Rules of Surds are:

1) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$	2) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Remember you can simplify surds and rationalise denominators.

Exercise A

1) Simplify

a). $\sqrt{8}$
b). $\sqrt{20}$
c). $\sqrt{98}$
d). $\sqrt{27}$
e). $\sqrt{48}$
f). $\sqrt{125}$
g). $\sqrt{50}$
h). $\sqrt{72}$
i). $\sqrt{128}$
j). $\sqrt{96}$
k). $\sqrt{216}$
l). $\sqrt{1210}$

2) Simplify

a). $\sqrt{8} + \sqrt{18}$
b). $\sqrt{45} - \sqrt{20}$
c). $\sqrt{50} + \sqrt{18}$
d). $\sqrt{98} - \sqrt{18}$
e). $\sqrt{48} + \sqrt{75}$
f). $\sqrt{245} - \sqrt{20}$
g). $4\sqrt{5} - \sqrt{20}$
h). $\sqrt{12} + 3\sqrt{75}$
i). $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$
j). $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$

3) Rationalise the denominator

a). $\frac{1}{\sqrt{3}}$
b). $\frac{5}{\sqrt{2}}$
c). $\frac{3}{\sqrt{3}}$
d). $\frac{16}{3\sqrt{2}}$
e). $\frac{14}{\sqrt{7}}$
f). $\sqrt{\left(\frac{5}{3}\right)}$
g). $\sqrt{\left(\frac{27}{50}\right)}$
h). $\frac{a}{\sqrt{a}}$
i). $\frac{a^2 b}{\sqrt{ab}}$
j). $\frac{\left(2-\sqrt{3}\right)}{\sqrt{2}}$
k). $\frac{\left(4-2\sqrt{3}\right)}{\sqrt{2}}$

Chapter 8: ALGRBRAIC FRACTIONS

Rules for algebraic fractions are the same as those for numerical fractions. You must remember these basic rules:

1. When adding and subtracting always find the **lowest** common denominator first

2. When multiplying and dividing always factorise first and cancel anything you can before you multiply. Remember with division you should swap the second fraction and multiply.

3. When cancelling ensure that you cancel with **all** terms. A term is separated by a '+' or '-' sign.

Exercise A

1) Simplify these algebraic fractions where possible, or write "cannot be simplified".

a. $\frac{5x+15}{5}$ **b.** $\frac{7}{14x+21}$ **c.** $\frac{x+2x}{5x}$ **d.** $\frac{2x+2}{2x-2}$ **e.** $\frac{3x+4}{4x-3}$

f. $\frac{ab}{a+b}$ **g.** $\frac{h^2}{h(x+y)}$ **h.** $\frac{6y}{2(y+1)}$ **i.** $\frac{x^2}{2x+xy}$ **j.** $\frac{x^3}{x^2+x}$

k. $\frac{a+b}{b} x \frac{a}{a+b}$ **l.** $\frac{a-b}{a} x \frac{a}{a+b}$ **m.** $\frac{2n^2+4n}{8n^3}$ **n.** $\frac{x+a}{x+a}$ **o.** $\frac{x+a}{x-a}$

2) Simplify the following expressions.

a). $\frac{x^2+7x+10}{4x^3} \times \frac{8x^2}{x^2+3x+2}$

d). $\frac{x^4-2x^3}{x+2} \times \frac{x^2+9x+14}{x^3+3x^2-10x}$

b). $\frac{a-b}{a} \times \frac{a^2}{a^2-b^2} \times \frac{a+b}{a}$

e). $\frac{(a+b)^2}{5a-5b} \times \frac{a-b}{3a+3b}$

c). $\frac{x^2-9}{x^2-1} \times \frac{x^2+2x-3}{x^2-2x-3}$

f). $\frac{x^2+x-2}{x^2+7x+12} \times \frac{x^2+5x+4}{x-1} \times \frac{2x+6}{x^2+6x+8}$

3) Express the following as single fractions.

a). $\frac{x+5}{4} - \frac{x-1}{5}$

b). $\frac{x-3}{3} + \frac{x+2}{2}$

c). $\frac{x-1}{2} - \frac{x-5}{3}$

d). $\frac{1}{x+1} + \frac{1}{x+2}$

e). $\frac{3}{x+4} - \frac{1}{x-1}$

f). $\frac{3}{2x-1} - \frac{4}{5x+2}$

g). $\frac{x^2}{x^2-5x+6} + \frac{4}{x-2}$

h). $\frac{x+1}{x^2-4x+3} - \frac{x-3}{x-1}$

Chapter 9 Sketching Curves and Transformations.

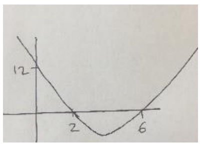
Curve Sketching
When we sketch curves of quadratics we look for important points on the graph, these are the roots, the y-intercept and the turning point.

To find the roots we let y = 0 and solve the quadratic.
To find the y-intercept we let x = 0 and solve.
To find the turning point we complete the square in the form $(x + a)^2 + b$, then the turning point becomes (-a, b)

Example:
Sketch the graph of $y = x^2 - 8x + 12$
Solution:
When y = 0, $x^2 - 8x + 12 = 0$, and solving gives us x = 6, or x = 2
When x = 0, $y = (0)^2 - 8(0) + 12$, hence y = 12.
Completing the square gives $y = (x - 4)^2 - 16 + 12$,
$y = (x - 4)^2 - 4$, hence the turning point is (4, -4).

Now that we have all the information we know that the graph cuts the x-axis at (2, 0) and (6, 0), cuts the y-axis at (0, 12) and has a turning point at (4, -4).

The resulting graph is



Graph Transformations
When we transform graphs we can either stretch or move the coordinates, each is dictated by a specific function transformation, if our original graph is f(x), then:

f(x) + a is a translation of the graph in the y-direction of $\binom{0}{a}$
f(x + a) is a translation of the graph in the x-direction of $\binom{-a}{0}$
bf(x) is a stretch of the graph in the y-direction scale factor b
f(bx) is a stretch of the graph in the x-direction scale factor $\frac{1}{b}$

Example:
A graph f(x) passes through coordinates (0, 6), (3, 9), (15,0)
Solution:

a) f(x) + 4
We are adding 4 to all 4 values here, so the coordinates become (0, 10), (3, 13), (15, 4)

b) f(3x)
We are multiplying all x values by $\frac{1}{3}$, so the coordinates become (0, 6), (1, 9), (5, 0)

c) 2f(x -3)
We are multiplying all y values by 2, and adding 3 to all x values,
so the coordinates become (3, 12), (6, 18), (18, 0)

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1. Sketch the following graphs, showing where the curves cut the coordinate axes and the coordinates of the maximum or minimum points.

a). $y = (x - 2)(x + 4)$

b). $y = (2 - x)(x - 8)$

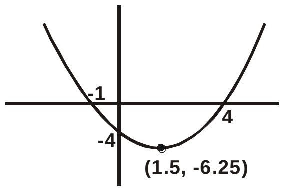
c). $y = x^2 - 3x - 10$

d). $y = 2x^2 + 10x - 7$

e). $y = (2x - 1)^2$

2. The graph of $y = f(x)$ is shown. The curves cuts the x axis at (–1, 0) and (4, 0) and the y axis at

(0, –4) as shown. The coordinates of the minimum point are (1.5, –6.25).



On separate axes, sketch the following curves, indicating on your sketches the coordinates of the minimum point and where possible the coordinates of where the curve intercepts the x and y axes.

a). $y = f(x) - 2$ b). $y = f(x - 2)$ c). $y = 2f(x)$

d). $y = f(2x)$ e). $y = -f(x)$ f). $y = f(-x)$

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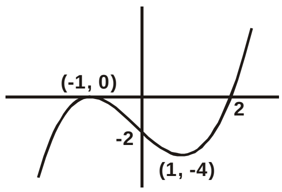
3. The graph of $y = g(x)$ is shown.



The coordinates of the maximum and minimum points are (–1, 0) and (1, –4), respectively. Additionally the curve crosses the x and y axes at (2, 0) and (0, –2), respectively.

Indicating where possible the coordinates of any intercepts of the curve with the axes and the coordinates of the maximum and minimum points, sketch the following curves.

a). $y = g(-x)$ b). $g(\frac{1}{2}x)$

c). $y = g(x - 2)$ d). $y = 3g(x)$

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4. a). You are given that

$$x^2 - 6x + 16 \equiv (x - p)^2 + q$$

Find the values of p and q.

b). Sketch the graph of

$$y = x^2 - 6x + 16$$

On your sketch indicate the coordinates of any intercepts with the x and y axes and the coordinates of the minimum point.

Describe geometrically the transformation that maps $y = x^2$ on to the graph of $y = x^2 - 6x + 16$.

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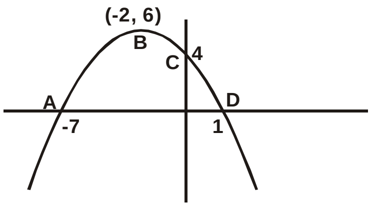
5. Sketch the curves

a). $y = (x - 2)^2$ b). $y = (x - 2)^2 + A$

In each case indicate the coordinates of the x and y intercepts and the coordinates of the minimum point.

6. The graph of y = f(x) is shown.



A, B, C and D are x and y intercepts and a maximum as appropriate.

a). On separate axes and, where possible, indicating the transformed coordinates of A, B, C and D sketch the following curves.

a). $y = f(-x)$ b). $y = f(x) - 3$
c). $y = f(2x)$ d). $y = 3f(x)$

SOLUTIONS TO THE EXERCISES

CHAPTER 1

Ex A 1) $(x-3)(x+2)$ 2) $(x+8)(x-2)$ 3) $(2x+1)(x+2)$ 4) $x(2x-3)$ 5) $(3x-1)(x+2)$

6) $(2y+3)(y+7)$ 7) $(7y-3)(y-1)$ 8) $5(2x-3)(x+2)$ 9) $(2x+5)(2x-5)$ 10) $(x-3)(x-y)$

11) $4(x-2)(x-1)$ 12) $(4m-9n)(4m+9n)$ 13) $y(2y-3a)(2y+3a)$ 14) $2(4x+5)(x-4)$

CHAPTER 2

Ex A 1) $t = \frac{32rP}{w}$ 2) $t = \pm\sqrt{\frac{32rP}{w}}$ 3) $t = \pm\sqrt{\frac{3V}{\pi h}}$ 4) $t = \frac{P^2 g}{2}$ 5) $t = v - \frac{Pag}{w}$ 6) $t = \pm\sqrt{\frac{r-a}{b}}$

Ex B 1) $x = \frac{c-3}{a-b}$ 2) $x = \frac{3a+2k}{k-3}$ 3) $x = \frac{2y+3}{5y-2}$ 4) $x = \frac{ab}{b-a}$

CHAPTER 3

1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2

3) a) -1/2, 4/3 b) 0.5, 2.5 4) a) -5.30, -1.70 b) 1.07, -0.699 c) -1.20, 1.45

d) no solutions e) no solutions f) no solutions

5) a). $x = 4 \pm \sqrt{21}$, b). $x = -3 \pm \sqrt{24}$ (or $2\sqrt{6}$) , c). $x = -4 \pm \sqrt{15}$, d). $x = 5 \pm \sqrt{26}$

CHAPTER 4

1. a). $b^2 - 4ac = -16$

As the discriminant is negative the original equation has no real roots.

b). $b^2 - 4ac = 0$

As the discriminant is zero the original equation has one distinct real root.

c). $b^2 - 4ac = 100$

As the discriminant is positive the original equation has two real roots.

d). $b^2 - 4ac = 216$

As the discriminant is positive the original equation has two real roots.

2. For equal roots $b^2 - 4ac = 0$, Hence $k = \pm 4$

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3. $k = \frac{3}{8}$

4. $b^2 - 4ac = -56$. As this is negative the original equation has no real solutions.

5. $m = -1$ or $m = -9$

6. a). If there are no real solutions $b^2 - 4ac < 0$.

So $k^2 - 4k < 0$

b) $0 < k < 4$

CHAPTER 5

1. a) Solutions are (4, 12) and (–1, –3).

b) Solutions are (1, –3) and (2, –2).

c) Solutions are (2, –1) and (3, 1).

d) Solutions are (1, 1) and ($\frac{1}{4}$, – $\frac{5}{4}$).

e). (2, –6), (–2, 6), (6, –2), (–6, 2).

f). Solutions are ($\frac{1}{2}$, $\frac{1}{3}$) and ($\frac{1}{6}$, $\frac{7}{9}$).

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2 Solutions are (– $\frac{1}{2}$, –2) and (2, 3)

3. Solutions are ($\frac{2}{5}$, $\frac{14}{5}$) and (–2, –2)

4. Solutions are (1, 1) and (–3, 9)

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5. Either using the discriminant to show there is one solution, or solving the simultaneous equations to give the only solution of (3, –1)

i.e. the line is a tangent to the circle at (3, –1)

6. Using the discriminant equal to zero, then solving gives solution (–2, 2)

7. The discriminant is negative. So there are no solutions.

CHAPTER 6

Ex A

1) $5b^6$ 2) $6c^7$ 3) b^3c^4 4) $-12n^8$ 5) $4n^5$ 6) d^2 7) a^6 8) $-d^{12}$

Ex B

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9 8) 9/4 9) ¼ 10) 0.2 11) 4/9 12) 64

13) $6a^3$ 14) x 15) xy^2

6) a) $\frac{1}{16}$ b) 1 c) $\frac{2}{3}$

CHAPTER 7

Ex A

1. a). $2\sqrt{2}$	b). $2\sqrt{5}$	c). $7\sqrt{2}$	d). $3\sqrt{3}$
e). $4\sqrt{2}$	f). $5\sqrt{5}$	g). $5\sqrt{2}$	h). $6\sqrt{2}$
i). $8\sqrt{2}$	j). $4\sqrt{6}$	k). $6\sqrt{6}$	l). $11\sqrt{10}$
2. a). $5\sqrt{2}$	b). $\sqrt{5}$	c). $8\sqrt{2}$	d). $4\sqrt{2}$
e). $9\sqrt{3}$	f). $5\sqrt{5}$	g). $2\sqrt{5}$	h). $17\sqrt{3}$
i). $\sqrt{2}$	j). $-8\sqrt{5}$	k). 0	
3. a). $\frac{\sqrt{3}}{3}$	b). $\frac{5\sqrt{2}}{2}$	c). $\sqrt{3}$	d). $\frac{8\sqrt{2}}{3}$

e). $2\sqrt{7}$ f). $\frac{\sqrt{15}}{3}$ g). $\frac{3\sqrt{6}}{10}$ h). $\sqrt{a}$

i). $a\sqrt{ab}$ j). $\frac{2\sqrt{2}-\sqrt{6}}{2}$ k). $2\sqrt{2} - \sqrt{6}$

CHAPTER 8

Ex A

1 a. $x + 3$ **b.** $\frac{1}{2x+3}$ **c.** $\frac{3}{5}$ **d.** $\frac{x+1}{x-1}$

e. c. b. s. **f.** c. b. s. **g.** $\frac{h}{x+y}$ **h.** $\frac{3y}{y+1}$

i. $\frac{x}{2+y}$ **j.** $\frac{x^2}{x+1}$ **k.** $\frac{a}{b}$ **l.** $\frac{a-b}{a+b}$

m. $\frac{n+2}{4n^2}$ **n.** 1 **o.** c. b. s.

2. a). $\frac{x+29}{20}$ **b).** $\frac{5x}{6}$ **c).** $\frac{x+7}{6}$ **d).** $\frac{2x+3}{(x+1)(x+2)}$ **e).** $\frac{2x-7}{(x+4)(x-1)}$ **f).** $\frac{7x+10}{(2x-1)(5x+2)}$

g). $\frac{x+6}{x-3}$ **h).** $\frac{7x-x^2-8}{(x-3)(x-1)}$

3. a). $\frac{2(x+5)}{x+1}$ **b).** 1

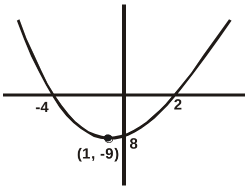
c). $\frac{(x+3)^2}{(x+1)^2}$ **d).** $\frac{x^2(x+7)}{x+5}$

e). $\frac{a+b}{15}$ **f).** $\frac{2(x+1)}{x+4}$

CHAPTER 9

1.

a).



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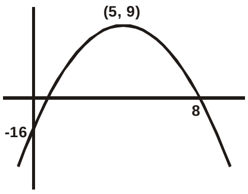

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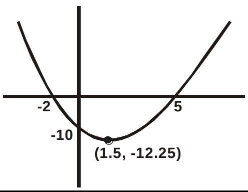

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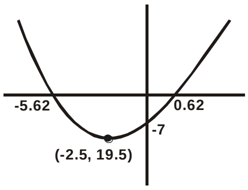

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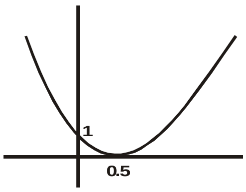
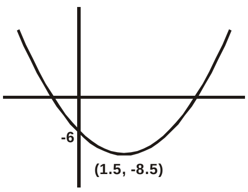


2.

a).



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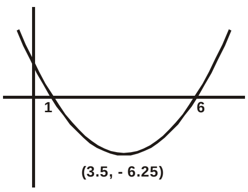

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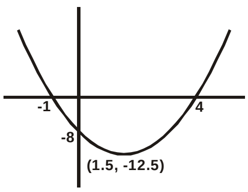
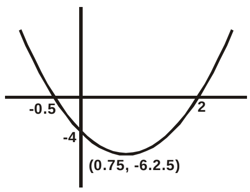
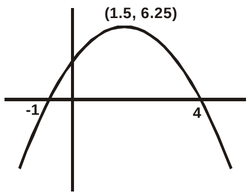
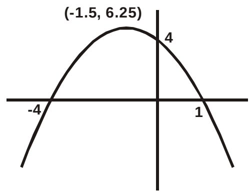


d).



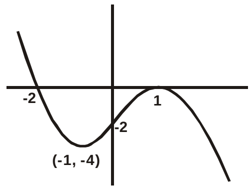
e). f).





3.

a).



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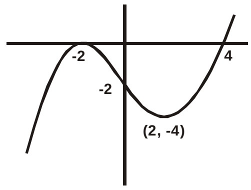

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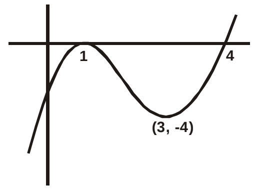
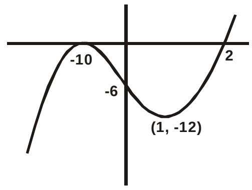


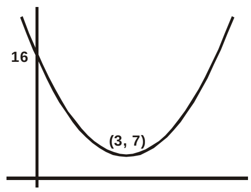
d).



4. a). $(x-3)^2 + 7$

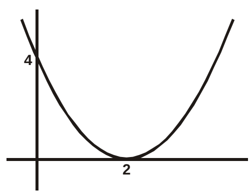
So the minimum is at (3, 7).

b). As the minimum value of the function is 7, the curve does not cross the x axis.

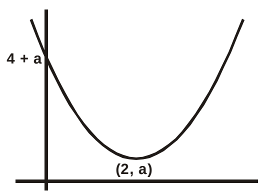


8.

a).

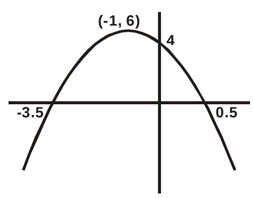


b).



9.

a).



b).